# Continuous Martingales And Brownian Motion Grundlehren Der Mathematischen Wissenschaften

Continuous Martingales and Brownian MotionSpringer Science & Business Media
This text is an introduction to the modern theory and applications of probability and stochastics. The style and coverage is geared towards the theory of stochastic processes, but with some attention to the applications. In many instances the gist of the problem is introduced in practical, everyday language and then is made precise in mathematical form. The first four chapters are on probability theory: measure and integration, probability spaces, conditional expectations, and the classical limit theorems. There follows chapters on martingales, Poisson random measures, Levy Processes, Brownian motion, and Markov Processes. Special attention is paid to Poisson random measures and their roles in regulating the excursions of Brownian motion and the jumps of Levy and Markov processes. Each chapter has a large number of varied examples and exercises. The book is based on the author’s lecture notes in courses offered over the years at Princeton University. These courses attracted graduate students from engineering, economics, physics, computer sciences, and mathematics. Erhan Cinlar has received many awards for excellence in teaching, including the President’s Award for Distinguished Teaching at Princeton University. His research interests include theories of Markov processes, point processes, stochastic calculus, and stochastic flows. The book is full of insights and observations that only a lifetime researcher in probability can have, all told in a lucid yet precise style.
Semimartingale Theory and Stochastic Calculus presents a systematic and detailed account of the general theory of stochastic processes, the semimartingale theory, and related stochastic calculus. The book emphasizes stochastic integration for semimartingales, characteristics of semimartingales, predictable representation properties and weak convergence of semimartingales. It also includes a concise treatment of absolute continuity and singularity, contiguity, and entire separation of measures by semimartingale approach. Two basic types of processes frequently encountered in applied probability and statistics are highlighted: processes with independent increments and marked point processes encountered frequently in applied probability and statistics. Semimartingale Theory and Stochastic Calculus is a self-contained and comprehensive book that will be valuable for research mathematicians, statisticians, engineers, and students.
This volume collects papers about the laws of geometric Brownian motions and their time-integrals, written by the author and coauthors between 1988 and 1998. Throughout the volume, connections with more recent studies involving exponential functionals of Lévy processes are indicated. Some papers originally published in French are made available in English for the first time.
The purpose, level, and style of this new edition conform to the tenets set forth in the original preface. The authors continue with their tack of developing simultaneously theory and applications, intertwined so that they refurbish and elucidate each other. The authors have made three main kinds of changes. First, they have enlarged on the topics treated in the first edition. Second, they have added many exercises and problems at the end of each chapter. Third, and most important, they have supplied, in new chapters, broad introductory discussions of several classes of stochastic processes not dealt with in the first edition, notably martingales, renewal and fluctuation phenomena associated with random sums, stationary stochastic processes, and diffusion theory.
Kiyosi Itô's greatest contribution to probability theory may be his introduction of stochastic differential equations to explain the Kolmogorov-Feller theory of Markov processes. Starting with the geometric ideas that guided him, this book gives an account of Itô's program. The modern theory of Markov processes was initiated by A. N. Kolmogorov. However, Kolmogorov's approach was too analytic to reveal the probabilistic foundations on which it rests. In particular, it hides the central role played by the simplest Markov processes: those with independent, identically distributed increments. To remedy this defect, Itô interpreted Kolmogorov's famous forward equation as an equation that describes the integral curve of a vector field on the space of probability measures. Thus, in order to show how Itô's thinking leads to his theory of stochastic integral equations, Stroock begins with an account of integral curves on the space of probability measures and then arrives at stochastic integral equations when he moves to a pathspace setting. In the first half of the book, everything is done in the context of general independent increment processes and without explicit use of Itô's stochastic integral calculus. In the second half, the author provides a systematic development of Itô's theory of stochastic integration: first for Brownian motion and then for continuous martingales. The final chapter presents Stratonovich's variation on Itô's theme and ends with an application to the characterization of the paths on which a diffusion is supported. The book should be accessible to readers who have mastered the essentials of modern probability theory and should provide such readers with a reasonably thorough introduction to continuous-time, stochastic processes.
The main purpose of the book is to present, at a graduate level and in a self-contained way, the most important aspects of the theory of continuous stochastic processes in continuous time and to introduce some of its ramifications such as the theory of semigroups, the Malliavin calculus, and the Lyons' rough paths. This book is intended for students, or even researchers, who wish to learn the basics in a concise but complete and rigorous manner. Several exercises are distributed throughout the text to test the understanding of the reader and each chapter ends with bibliographic comments aimed at those interested in exploring the materials further. Stochastic calculus was developed in the 1950s and the range of its applications is huge and still growing today. Besides being a fundamental component of modern probability theory, domains of applications include but are not limited to: mathematical finance, biology, physics, and engineering sciences. The first part of the text is devoted to the general theory of stochastic processes. The author focuses on the existence and regularity results for processes and on the theory of martingales. This allows him to

introduce the Brownian motion quickly and study its most fundamental properties. The second part deals with the study of Markov processes, in particular, diffusions. The author's goal is to stress the connections between these processes and the theory of evolution semigroups. The third part deals with stochastic integrals, stochastic differential equations and Malliavin calculus. In the fourth and final part, the author presents an introduction to the very new theory of rough paths by Terry Lyons.

Initially the theory of convergence in law of stochastic processes was developed quite independently from the theory of martingales, semimartingales and stochastic integrals. Apart from a few exceptions essentially concerning diffusion processes, it is only recently that the relation between the two theories has been thoroughly studied. The authors of this Grundlehren volume, two of the international leaders in the field, propose a systematic exposition of convergence in law for stochastic processes, from the point of view of semimartingale theory, with emphasis on results that are useful for mathematical theory and mathematical statistics. This leads them to develop in detail some particularly useful parts of the general theory of stochastic processes, such as martingale problems, and absolute continuity or contiguity results. The book contains an elementary introduction to the main topics: theory of martingales and stochastic integrales, Skorokhod topology, etc., as well as a large number of results which have never appeared in book form, and some entirely new results. It should be useful to the professional probabilist or mathematical statistician, and of interest also to graduate students.

Stochastic processes are necessary ingredients for building models of a wide variety of phenomena exhibiting time varying randomness. This text offers easy access to this fundamental topic for many students of applied sciences at many levels. It includes examples, exercises, applications, and computational procedures. It is uniquely useful for beginners and non-beginners in the field. No knowledge of measure theory is presumed.

Emphasizing fundamental mathematical ideas rather than proofs, Introduction to Stochastic Processes, Second Edition provides quick access to important foundations of probability theory applicable to problems in many fields. Assuming that you have a reasonable level of computer literacy, the ability to write simple programs, and the access to software for linear algebra computations, the author approaches the problems and theorems with a focus on stochastic processes evolving with time, rather than a particular emphasis on measure theory. For those lacking in exposure to linear differential and difference equations, the author begins with a brief introduction to these concepts. He proceeds to discuss Markov chains, optimal stopping, martingales, and Brownian motion. The book concludes with a chapter on stochastic integration. The author supplies many basic, general examples and provides exercises at the end of each chapter. New to the Second Edition: Expanded chapter on stochastic integration that introduces modern mathematical finance Introduction of Girsanov transformation and the Feynman-Kac formula Expanded discussion of Itô's formula and the Black-Scholes formula for pricing options New topics such as Doob's maximal inequality and a discussion on self similarity in the chapter on Brownian motion Applicable to the fields of mathematics, statistics, and engineering as well as computer science, economics, business, biological science, psychology, and engineering, this concise introduction is an excellent resource both for students and professionals.

The most fundamental concepts in the theory of stochastic processes are the Markov property and the martingale property. This book is written for those who are familiar with both of these ideas in the discrete-time setting, and who now wish to explore stochastic processes in the continuous-time context. It has been our goal to write a systematic and thorough exposition of this subject, leading in many instances to the frontiers of knowledge. At the same time, we have endeavored to keep the mathematical prerequisities as low as possible, namely, knowledge of measure-theoretic probability and some acquaintance with discrete-time processes. The vehicle we have chosen for this task is Brownian motion, which we present as the canonical example of both a Markov process and a martingale in continuous time. We support this point of view by showing how by means of stochastic integration and random time change, all continuous martingales and a multitude of continuous Markov processes can be represented in terms of Brownian motion. This approach forces us to leave aside those processes which do not have continuous paths. Thus, the Poisson process is not a primary object of study, although it is developed to be used as a tool when we later study passage times of Brownian motion.

This book sheds new light on stochastic calculus, the branch of mathematics that is most widely applied in financial engineering and mathematical finance. The first book to introduce pathwise formulae for the stochastic integral, it provides a simple but rigorous treatment of the subject, including a range of advanced topics. The book discusses in-depth topics such as quadratic variation, Ito formula, and Emery topology. The authors briefly address continuous semi-martingales to obtain growth estimates and study solution of a stochastic differential equation (SDE) by using the technique of random time change. Later, by using Metivier–Pellaumail inequality, the solutions to SDEs driven by general semi-martingales are discussed. The connection of the theory with mathematical finance is briefly discussed and the book has extensive treatment on the representation of martingales as stochastic integrals and a second fundamental theorem of asset pricing. Intended for undergraduate- and beginning graduate-level students in the engineering and mathematics disciplines, the book is also an excellent reference resource for applied mathematicians and statisticians looking for a review of the topic.

Markov processes are among the most important stochastic processes for both theory and applications. This book develops the general theory of these processes, and applies this theory to various special examples. The initial chapter is devoted to the most important classical example - one dimensional Brownian motion. This, together with a chapter on continuous time Markov chains, provides the motivation for the general setup based on semigroups and generators. Chapters on stochastic calculus and probabilistic potential theory give an introduction to some of the key areas of application of Brownian motion and its relatives. A chapter on interacting particle systems treats a more recently developed class of Markov processes that have as their origin problems in physics and biology. This is a textbook for a graduate course that can follow one that covers basic probabilistic limit theorems and discrete time processes.

In three chapters on Exponential Martingales, BMO-martingales, and Exponential of BMO, this book explains in detail the beautiful properties of continuous exponential martingales that play an essential role in various questions concerning the absolute continuity of probability laws of stochastic processes. The second and principal aim is to provide a full report on the exciting results on BMO in the theory of exponential martingales. The reader is assumed to be familiar with the general theory of continuous martingales.

Unique for its broad and yet comprehensive coverage of modern probability theory, ranging from first principles and standard textbook material to more advanced topics. In spite of the economical exposition, careful proofs are provided for all main results. After a detailed discussion of classical limit theorems, martingales, Markov chains, random walks, and stationary processes, the author moves on to a modern treatment of Brownian motion, L=82vy processes, weak convergence, It=93 calculus, Feller processes, and SDEs. The more advanced parts include material on local time, excursions, and additive functionals, diffusion processes, PDEs and potential theory, predictable processes, and general semimartingales. Though primarily intended as a general reference for researchers and graduate students in probability theory and related areas of analysis, the book is also suitable as a text for graduate and seminar courses on all levels, from elementary to advanced. Numerous easy to more challenging exercises are provided, especially for the early chapters. From the author of "Random Measures".

It has been 15 years since the first edition of Stochastic Integration and Differential Equations, A New Approach appeared, and in those years many other texts on the same subject have been published, often with connections to applications, especially mathematical finance. Yet in spite of the apparent simplicity of approach, none of these books has used the functional analytic method of presenting semimartingales and stochastic integration. Thus a 2nd edition seems worthwhile and timely, though it is no longer appropriate to call it "a new approach". The new edition has several significant changes, most prominently the addition of exercises for solution. These are intended to supplement the text, but lemmas needed in a proof are never relegated to the exercises. Many of the exercises have been tested by graduate students at Purdue and Cornell Universities. Chapter 3 has been completely redone, with a new, more intuitive and simultaneously elementary proof of the fundamental Doob-Meyer decomposition theorem, the more general version of the Girsanov theorem due to Lenglart, the Kazamaki-Novikov criteria for exponential local martingales to be martingales, and a modern treatment of compensators. Chapter 4 treats sigma martingales (important in finance theory) and gives a more comprehensive treatment of martingale representation, including both the Jacod-Yor theory and Emery’s examples of martingales that actually have martingale representation (thus going beyond the standard cases of Brownian motion and the compensated Poisson process). New topics added include an introduction to the theory of the expansion of filtrations, a treatment of the Fefferman martingale inequality, and that the dual space of the martingale space H^1 can be identified with BMO martingales. Solutions to selected exercises are available at the web site of the author, with current URL http://www.orie.cornell.edu/~protter/books.html.

Brownian motion is one of the most important stochastic processes in continuous time and with continuous state space. Within the realm of stochastic processes, Brownian motion is at the intersection of Gaussian processes, martingales, Markov processes, diffusions and random fractals, and it has influenced the study of these topics. Its central position within mathematics is matched by numerous applications in science, engineering and mathematical finance. Often textbooks on probability theory cover, if at all, Brownian motion only briefly. On the other hand, there is a considerable gap to more specialized texts on Brownian motion which is not so easy to overcome for the novice. The authors’ aim was to write a book which can be used as an introduction to Brownian motion and stochastic calculus, and as a first course in continuous-time and continuous-state Markov processes. They also wanted to have a text which would be both a readily accessible mathematical back-up for contemporary applications (such as mathematical finance) and a foundation to get easy access to advanced monographs. This textbook, tailored to the needs of graduate and advanced undergraduate students, covers Brownian motion, starting from its elementary properties, certain distributional aspects, path properties, and leading to stochastic calculus based on Brownian motion. It also includes numerical recipes for the simulation of Brownian motion.

This monograph discusses the existence and regularity properties of local times associated to a continuous semimartingale, as well as excursion theory for Brownian paths. Realizations of Brownian excursion processes may be translated in terms of the realizations of a Wiener process under certain conditions. With this aim in mind, the monograph presents applications to topics which are not usually treated with the same tools, e.g.: arc sine law, laws of functionals of Brownian motion, and the Feynman-Kac formula.

A graduate-course text, written for readers familiar with measure-theoretic probability and discrete-time processes, wishing to explore stochastic processes in continuous time. The vehicle chosen for this exposition is Brownian motion, which is presented as the canonical example of both a martingale and a Markov process with continuous paths. In this context, the theory of stochastic integration and stochastic calculus is developed, illustrated by results concerning representations of martingales and change of measure on Wiener space, which in turn permit a presentation of recent advances in financial economics. The book contains a detailed discussion of weak and strong solutions of stochastic differential equations and a study of local time for semimartingales, with special emphasis on the theory of Brownian local time. The whole is backed by a large number of problems and exercises.

Provides graduate students and practitioners in physics and economics with a better understanding of stochastic processes.

"This is a magnificent book! Its purpose is to describe in considerable detail a variety of techniques used by probabilists in the investigation of problems concerning Brownian motion....This is THE book for a capable graduate student starting out on research in probability: the effect of working through it is as if the authors are sitting beside one, enthusiastically explaining the theory, presenting further developments as exercises." –BULLETIN OF THE L.M.S.

The following notes represent approximately the second half of the lectures I gave in the Nachdiplomvorlesung, in ETH, Zurich, between October 1991 and February 1992, together with the contents of six additional lectures I gave in ETH, in November and December 1993. Part I, the elder brother of the present book [Part II], aimed at the computation, as explicitly as possible, of a number of interesting functionals of Brownian motion. It may be natural that Part II, the younger brother, looks more into the main technique with which Part I was "working", namely: martingales and stochastic calculus. As F. Knight writes, in a review article on Part I, in which research on Brownian motion is compared to gold mining: "In the days of P. Levy, and even as late as the theorems of "Ray and Knight" (1963), it was possible for the practiced eye to

pick up valuable reward without the aid of much technology . . . Thereafter, however, the rewards are increasingly achieved by the application of high technology". Although one might argue whether this golden age is really foregone, and discuss the "height" of the technology involved, this quotation is closely related to the main motivations of Part II: this technology, which includes stochastic calculus for general discontinuous semi-martingales, enlargement of filtrations, . . .
This monograph is an introduction to some aspects of stochastic analysis in the framework of normal martingales, in both discrete and continuous time. The text is mostly self-contained, except for Section 5.7 that requires some background in geometry, and should be accessible to graduate students and researchers having already received a basic training in probability. Prereq- sites are mostly limited to a knowledge of measure theory and probability, namely?-algebras,expectations,andconditionalexpectations.Ashortint- duction to stochastic calculus for continuous and jump processes is given in Chapter 2 using normal martingales, whose predictable quadratic variation is the Lebesgue measure. There already exists several books devoted to stochastic analysis for c- tinuous di?usion processes on Gaussian and Wiener spaces, cf. e.g. [51], [63], [65], [72], [83], [84], [92], [128], [134], [143], [146], [147]. The particular f- ture of this text is to simultaneously consider continuous processes and jump processes in the uni?ed framework of normal martingales.
Interactive particle systems is a branch of probability theory with close connections to mathematical physics and mathematical biology. This book takes three of the most important models in the area, and traces advances in our understanding of them since 1985. It explains and develops many of the most useful techniques in the field.
This book concentrates on some general facts and ideas of the theory of stochastic processes. The topics include the Wiener process, stationary processes, infinitely divisible processes, and Ito stochastic equations. Basics of discrete time martingales are also presented and then used in one way or another throughout the book. Another common feature of the main body of the book is using stochastic integration with respect to random orthogonal measures. In particular, it is used for spectral representation of trajectories of stationary processes and for proving that Gaussian stationary processes with rational spectral densities are components of solutions to stochastic equations. In the case of infinitely divisible processes, stochastic integration allows for obtaining a representation of trajectories through jump measures. The Ito stochastic integral is also introduced as a particular case of stochastic integrals with respect to random orthogonal measures. Although it is not possible to cover even a noticeable portion of the topics listed above in a short book, it is hoped that after having followed the material presented here, the reader will have acquired a good understanding of what kind of results are available and what kind of techniques are used to obtain them. With more than 100 problems included, the book can serve as a text for an introductory course on stochastic processes or for independent study. Other works by this author published by the AMS include, Lectures on Elliptic and Parabolic Equations in Holder Spaces and Introduction to the Theory of Diffusion Processes.
From the reviews: "Here is a momumental work by Doob, one of the masters, in which Part 1 develops the potential theory associated with Laplace's equation and the heat equation, and Part 2 develops those parts (martingales and Brownian motion) of stochastic process theory which are closely related to Part 1". --G.E.H. Reuter in Short Book Reviews (1985)
Now available in paperback for the first time; essential reading for all students of probability theory.
Stochastic calculus has important applications to mathematical finance. This book will appeal to practitioners and students who want an elementary introduction to these areas. From the reviews: "As the preface says, ‘This is a text with an attitude, and it is designed to reflect, wherever possible and appropriate, a prejudice for the concrete over the abstract’. This is also reflected in the style of writing which is unusually lively for a mathematics book."
--ZENTRALBLATT MATH
A highly readable introduction to stochastic integration and stochastic differential equations, this book combines developments of the basic theory with applications. It is written in a style suitable for the text of a graduate course in stochastic calculus, following a course in probability. Using the modern approach, the stochastic integral is defined for predictable integrands and local martingales; then It’s change of variable formula is developed for continuous martingales. Applications include a characterization of Brownian motion, Hermite polynomials of martingales, the Feynman–Kac functional and the Schrödinger equation. For Brownian motion, the topics of local time, reflected Brownian motion, and time change are discussed. New to the second edition are a discussion of the Cameron–Martin–Girsanov transformation and a final chapter which provides an introduction to stochastic differential equations, as well as many exercises for classroom use. This book will be a valuable resource to all mathematicians, statisticians, economists, and engineers employing the modern tools of stochastic analysis. The text also proves that stochastic integration has made an important impact on mathematical progress over the last decades and that stochastic calculus has become one of the most powerful tools in modern probability theory. —Journal of the American Statistical Association An attractive text...written in [a] lean and precise style...eminently readable. Especially pleasant are the care and attention devoted to details... A very fine book. —Mathematical Reviews
Following the publication of the Japanese edition of this book, several inter esting developments took place in the area. The author wanted to describe some of these, as well as to offer suggestions concerning future problems which he hoped would stimulate readers working in this field. For these reasons, Chapter 8 was added. Apart from the additional chapter and a few minor changes made by the author, this translation closely follows the text of the original Japanese edition. We would like to thank Professor J. L. Doob for his helpful comments on the English edition. T. Hida T. P. Speed v Preface The physical phenomenon described by Robert Brown was the complex and erratic motion of grains of pollen suspended in a liquid. In the many years which have passed since this description, Brownian motion has become an object of study in pure as well as applied mathematics. Even now many of its important properties are being discovered,

and doubtless new and useful aspects remain to be discovered. We are getting a more and more intimate understanding of Brownian motion.

This book offers a rigorous and self-contained presentation of stochastic integration and stochastic calculus within the general framework of continuous semimartingales. The main tools of stochastic calculus, including Itô's formula, the optional stopping theorem and Girsanov's theorem, are treated in detail alongside many illustrative examples. The book also contains an introduction to Markov processes, with applications to solutions of stochastic differential equations and to connections between Brownian motion and partial differential equations. The theory of local times of semimartingales is discussed in the last chapter. Since its invention by Itô, stochastic calculus has proven to be one of the most important techniques of modern probability theory, and has been used in the most recent theoretical advances as well as in applications to other fields such as mathematical finance. Brownian Motion, Martingales, and Stochastic Calculus provides a strong theoretical background to the reader interested in such developments. Beginning graduate or advanced undergraduate students will benefit from this detailed approach to an essential area of probability theory. The emphasis is on concise and efficient presentation, without any concession to mathematical rigor. The material has been taught by the author for several years in graduate courses at two of the most prestigious French universities. The fact that proofs are given with full details makes the book particularly suitable for self-study. The numerous exercises help the reader to get acquainted with the tools of stochastic calculus.

This book provides a comprehensive introduction to the theory of stochastic calculus and some of its applications. It is the only textbook on the subject to include more than two hundred exercises with complete solutions. After explaining the basic elements of probability, the author introduces more advanced topics such as Brownian motion, martingales and Markov processes. The core of the book covers stochastic calculus, including stochastic differential equations, the relationship to partial differential equations, numerical methods and simulation, as well as applications of stochastic processes to finance. The final chapter provides detailed solutions to all exercises, in some cases presenting various solution techniques together with a discussion of advantages and drawbacks of the methods used. Stochastic Calculus will be particularly useful to advanced undergraduate and graduate students wishing to acquire a solid understanding of the subject through the theory and exercises. Including full mathematical statements and rigorous proofs, this book is completely self-contained and suitable for lecture courses as well as self-study.

This book describes how neural networks operate from the mathematical point of view. As a result, neural networks can be interpreted both as function universal approximators and information processors. The book bridges the gap between ideas and concepts of neural networks, which are used nowadays at an intuitive level, and the precise modern mathematical language, presenting the best practices of the former and enjoying the robustness and elegance of the latter. This book can be used in a graduate course in deep learning, with the first few parts being accessible to senior undergraduates. In addition, the book will be of wide interest to machine learning researchers who are interested in a theoretical understanding of the subject.

Now available in paperback, this celebrated book has been prepared with readers' needs in mind, remaining a systematic guide to a large part of the modern theory of Probability, whilst retaining its vitality. The authors' aim is to present the subject of Brownian motion not as a dry part of mathematical analysis, but to convey its real meaning and fascination. The opening, heuristic chapter does just this, and it is followed by a comprehensive and self-contained account of the foundations of theory of stochastic processes. Chapter 3 is a lively and readable account of the theory of Markov processes. Together with its companion volume, this book helps equip graduate students for research into a subject of great intrinsic interest and wide application in physics, biology, engineering, finance and computer science.

This eagerly awaited textbook covers everything the graduate student in probability wants to know about Brownian motion, as well as the latest research in the area. Starting with the construction of Brownian motion, the book then proceeds to sample path properties like continuity and nowhere differentiability. Notions of fractal dimension are introduced early and are used throughout the book to describe fine properties of Brownian paths. The relation of Brownian motion and random walk is explored from several viewpoints, including a development of the theory of Brownian local times from random walk embeddings. Stochastic integration is introduced as a tool and an accessible treatment of the potential theory of Brownian motion clears the path for an extensive treatment of intersections of Brownian paths. An investigation of exceptional points on the Brownian path and an appendix on SLE processes, by Oded Schramm and Wendelin Werner, lead directly to recent research themes.

This textbook introduces the theory of stochastic processes, that is, randomness which proceeds in time. Using concrete examples like repeated gambling and jumping frogs, it presents fundamental mathematical results through simple, clear, logical theorems and examples. It covers in detail such essential material as Markov chain recurrence criteria, the Markov chain convergence theorem, and optional stopping theorems for martingales. The final chapter provides a brief introduction to Brownian motion, Markov processes in continuous time and space, Poisson processes, and renewal theory.Interspersed throughout are applications to such topics as gambler's ruin probabilities, random walks on graphs, sequence waiting times, branching processes, stock option pricing, and Markov Chain Monte Carlo (MCMC) algorithms.The focus is always on making the theory as well-motivated and accessible as possible, to allow students and readers to learn this fascinating subject as easily and painlessly as possible.

This textbook offers an approachable introduction to stochastic processes that explores the four pillars of random walk, branching processes, Brownian motion, and martingales. Building from simple examples, the authors focus on developing context and intuition before formalizing the theory of each topic. This inviting approach illuminates the key ideas and computations in the proofs, forming an ideal basis for further study. Consisting of many short chapters, the book begins with a comprehensive account of the simple random walk in one dimension. From here, different paths may be chosen according to interest. Themes span Poisson processes, branching processes, the Kolmogorov–Chentsov theorem, martingales, renewal theory, and Brownian motion. Special topics follow, showcasing a selection of important contemporary applications, including mathematical finance, optimal stopping, ruin theory, branching random walk, and equations of fluids. Engaging exercises accompany the theory throughout. Random Walk, Brownian Motion, and Martingales is an ideal introduction to the rigorous study of stochastic processes. Students and instructors alike will appreciate the accessible, example-driven approach. A single, graduate-level course in probability is assumed.

This compact yet thorough text zeros in on the parts of the theory that are particularly relevant to applications . It begins with a description of Brownian motion and the associated stochastic calculus, including their relationship to partial differential equations. It solves stochastic differential equations by a variety of methods and studies in detail the one-dimensional case. The book concludes with a treatment of semigroups and generators, applying the theory of Harris chains to diffusions, and presenting a quick course in weak convergence of Markov chains to diffusions. The presentation is unparalleled in its clarity and simplicity. Whether your students are interested in probability, analysis, differential geometry or applications in operations research, physics, finance, or the many other areas to which the subject applies, you'll find that this text brings together the material you need to effectively and efficiently impart the practical background they need.