# Introduction To Chaotic Dynamical Systems Solutions Manual

This book offers a complete and systematic presentation of chaotic billiards, with full proofs, exercises and illustrations. It is designed to give readers an understanding of crucial stochastic properties of chaotic billiards, including ergodicity, decay of correlations and the Central Limit Theorem. No other book offers a self-contained exposition of this new, dynamic and complex theory.

This book is conceived as a comprehensive and detailed text-book on non-linear dynamical systems with particular emphasis on the exploration of chaotic phenomena. The self-contained introductory presentation is addressed both to those who wish to study the physics of chaotic systems and non-linear dynamics intensively as well as those who are curious to learn more about the fascinating world of chaotic phenomena. Basic concepts like Poincaré section, iterated mappings, Hamiltonian chaos and KAM theory, strange attractors, fractal dimensions, Lyapunov exponents, bifurcation theory, self-similarity and renormalisation and transitions to chaos are thoroughly explained. To facilitate comprehension, mathematical concepts and tools are introduced in short sub-sections. The text is supported by numerous computer experiments and a multitude of graphical illustrations and colour plates emphasising the geometrical and topological characteristics of the underlying dynamics. This volume is a completely revised and enlarged second edition which comprises recently obtained research results of topical interest, and has been extended to include a new section on the basic concepts of probability theory. A completely new chapter on fully developed turbulence presents the successes of chaos theory, its limitations as well as future trends in the development of complex spatio-temporal structures. "This book will be of valuable help for my lectures" Hermann Haken, Stuttgart "This text-book should not be missing in any introductory lecture on non-linear systems and deterministic chaos" Wolfgang Kinzel, Würzburg “This well written book represents a comprehensive treatise on dynamical systems. It may serve as reference book for the whole field of nonlinear and chaotic systems and reports in a unique way on scientific developments of recent decades as well as important applications.” Joachim Peinke, Institute of Physics, Carl-von-Ossietzky University Oldenburg, Germany

An introduction to nonlinear differential equations which equips undergraduate students with the know-how to appreciate stability theory and bifurcation.

This rigorous undergraduate introduction to dynamical systems is an accessible guide for mathematics students advancing from calculus.

This book describes systematic design techniques for chaotic and hyperchaotic systems, the transition from one to the other, and their implementation in electronic circuits. It also discusses the collective phenomena manifested by these systems when connected by a physical coupling scheme. Readers will be introduced to collective behaviours, such as synchronization and oscillation suppression, and will learn how to implement nonlinear differential equations in electronic circuits. Further, the book shows how the choice of nonlinearity can lead to chaos and hyperchaos, even in a first-order time-delayed system. The occurrence of these phenomena, together with the efficiency of the design techniques described, is presented with theoretical studies, numerical characterization and experimental demonstrations with the corresponding electronic circuits, helping readers grasp the design aspects of dynamical systems as a whole in electronic circuits. The authors then discuss the usefulness of an active all-pass filter as the delay element, supported by their own experimental observations, as well as theoretical and numerical results. Including detailed analysis, as well as computations with suitable dedicated software packages, the book will be of interest to all academics and researchers who wish to expand their knowledge of the subtlety of nonlinear time-delayed systems. It also offers a valuable source of information for engineers, linking the design techniques of chaotic time-delayed systems with their collective phenomena.

A timely, accessible introduction to the mathematics ofchaos. The past three decades have seen dramatic developments in thetheory of dynamical systems, particularly regarding the explorationof chaotic behavior. Complex patterns of even simple processesarising in biology, chemistry, physics, engineering, economics, anda host of other disciplines have been investigated, explained, andutilized. Introduction to Discrete Dynamical Systems and Chaos makes theseexciting and important ideas accessible to students and scientistsby assuming, as a background, only the standard undergraduatetraining in calculus and linear algebra. Chaos is introduced at theoutset and is then incorporated as an integral part of the theoryof discrete dynamical systems in one or more dimensions. Both phasespace and parameter space analysis are developed with ampleexercises, more than 100 figures, and important practical examplessuch as the dynamics of atmospheric changes and neuralnetworks. An appendix provides readers with clear guidelines on how to useMathematica to explore discrete dynamical systems numerically.Selected programs can also be downloaded from a Wiley ftp site(address in preface). Another appendix lists possible projects thatcan be assigned for classroom investigation. Based on the author's1993 book, but boasting at least 60% new, revised, and updatedmaterial, the present Introduction to Discrete Dynamical Systemsand Chaos is a unique and extremely useful resource for allscientists interested in this active and intensely studiedfield. An Instructor's Manual presenting detailed solutions to all theproblems in the book is available upon request from the Wileyeditorial department.

Basic concepts Zero-dimensional dynamics One-dimensional dynamics Two-dimensional dynamics Systems with 1.5 degrees of freedom Systems generated by three-dimensional vector fields Lyapunov exponents Appendix Bibliography Index.

This textbook is aimed at newcomers to nonlinear dynamics and chaos, especially students taking a first course in the subject. The presentation stresses analytical methods, concrete examples, and geometric intuition. The theory is developed systematically, starting with first-order differential equations and their bifurcations, followed by phase plane analysis, limit cycles and their bifurcations, and culminating with the Lorenz equations, chaos, iterated maps, period doubling, renormalization, fractals, and strange attractors.

Provides a new and more realistic framework for describing the dynamics of non-linear systems. A number of issues arising in applied dynamical systems from the viewpoint of problems of phase space transport are raised in this monograph. Illustrating phase space transport problems arising in a variety of applications that can be modeled as time-periodic perturbations of planar Hamiltonian systems, the book begins with the study of transport in the associated two-dimensional Poincaré Map. This serves as a starting point for the further motivation of the transport issues through the development of ideas in a non-perturbative framework with generalizations to higher dimensions as well as more general time dependence. A timely and important contribution to those concerned with the applications of mathematics.

The study of nonlinear dynamical systems has exploded in the past 25 years, and Robert L. Devaney has made these advanced research developments accessible to undergraduate and graduate mathematics students as well as researchers in other disciplines with the introduction of this widely praised book. In this second edition of his best-selling text, Devaney includes new material on the orbit diagram fro maps of the interval and the Mandelbrot set, as well as striking color photos illustrating both Julia and Mandelbrot sets. This book assumes no prior acquaintance with advanced mathematical topics such as measure theory, topology, and differential geometry. Assuming only a knowledge of calculus, Devaney introduces many of the basic concepts of modern dynamical systems theory and leads the reader to the point of current research in several areas.

Differential Equations, Dynamical Systems, and an Introduction to Chaos, Second Edition, provides a rigorous yet accessible introduction to differential equations and dynamical systems. The original text by three of the world's leading mathematicians has become the standard textbook for graduate courses in this area. Thirty years in the making, this Second Edition brings students to the brink of contemporary research, starting from a background that includes only calculus and elementary linear algebra. The book explores the dynamical aspects of ordinary differential equations and the relations between dynamical systems and certain fields outside pure mathematics. It presents the simplification of many theorem hypotheses and includes bifurcation theory throughout. It contains many new figures and illustrations; a simplified treatment of linear algebra; detailed discussions of the chaotic behavior in the Lorenz attractor, the Shil'nikov systems, and the double scroll attractor; and increased coverage of discrete dynamical systems. This book will be particularly useful to advanced students and practitioners in higher mathematics. * Developed by award-winning researchers and authors * Provides a rigorous yet accessible introduction to differential equations and dynamical systems * Includes bifurcation theory throughout * Contains numerous explorations for students to embark upon NEW IN THIS EDITION * New contemporary material and updated applications * Revisions throughout the text, including simplification of many theorem hypotheses * Many new figures and illustrations * Simplified treatment of linear algebra * Detailed discussion of the chaotic behavior in the Lorenz attractor, the Shil'nikov systems, and the double scroll attractor * Increased coverage of discrete dynamical systems

Hirsch, Devaney, and Smale’s classic Differential Equations, Dynamical Systems, and an Introduction to Chaos has been used by professors as the primary text for undergraduate and graduate level courses covering differential equations. It provides a theoretical approach to dynamical systems and chaos written for a diverse student population among the fields of mathematics, science, and engineering. Prominent experts provide everything students need to know about dynamical systems as students seek to develop sufficient mathematical skills to analyze the types of differential equations that arise in their area of study. The authors provide rigorous exercises and examples clearly and easily by slowly introducing linear systems of differential equations. Calculus is required as specialized advanced topics not usually found in elementary differential equations courses are included, such as exploring the world of discrete dynamical systems and describing chaotic systems. Classic text by three of the world’s most prominent mathematicians Continues the tradition of expository excellence Contains updated material and expanded applications for use in applied studies

Over the last four decades there has been extensive development in the theory of dynamical systems. This book aims at a wide audience where the first four chapters have been used for an undergraduate course in Dynamical Systems. Material from the last two chapters and from the appendices has been used quite a lot for master and PhD courses. All chapters are concluded by an exercise section. The book is also directed towards researchers, where one of the challenges is to help applied researchers acquire background for a better understanding of the data that computer simulation or experiment may provide them with the development of the theory.

This book provides a broad introduction to the subject of dynamical systems, suitable for a one- or two-semester graduate course. In the first chapter, the authors introduce over a dozen examples, and then use these examples throughout the book to motivate and clarify the development of the theory. Topics include topological dynamics, symbolic dynamics, ergodic theory, hyperbolic dynamics, one-dimensional dynamics, complex dynamics, and measure-theoretic entropy. The authors top off the presentation with some beautiful and remarkable applications of dynamical systems to such areas as number theory, data storage, and Internet search engines. This book grew out of lecture notes from the graduate dynamical systems course at the University of Maryland, College Park, and reflects not only the tastes of the authors, but also to some extent the collective opinion of the Dynamics Group at the University of Maryland, which includes experts in virtually every major area of dynamical systems.

Chaos is the idea that a system will produce very different long-term behaviors when the initial conditions are perturbed only slightly. Chaos is used for novel, time- or energy-critical interdisciplinary applications. Examples include high-performance circuits and devices, liquid mixing, chemical reactions, biological systems, crisis management, secure information processing, and critical decision-making in politics, economics, as well as military applications, etc. This book presents the latest investigations in the theory of chaotic systems and their dynamics. The book covers some theoretical aspects of the subject arising in the study of both discrete and continuous-time chaotic dynamical systems. This book presents the state-of-the-art of the more advanced studies of chaotic dynamical systems.

A First Course in Chaotic Dynamical Systems: Theory and Experiment is the first book to introduce modern topics in dynamical systems at the undergraduate level. Accessible to readers with only a background in calculus, the book integrates both theory and computer experiments into its coverage of contemporary ideas in dynamics. It is designed as a gradual introduction to the basic mathematical ideas behind such topics as chaos, fractals, Newton's method, symbolic dynamics, the Julia set, and the Mandelbrot set, and includes biographies of some of the leading researchers in the field of dynamical systems. Mathematical and computer experiments are integrated throughout the text to help illustrate the meaning of the theorems presented. Chaotic

Dynamical Systems Software, Labs 1-6 is a supplementary labouratory software package, available separately, that allows a more intuitive understanding of the mathematics behind dynamical systems theory. Combined with A First Course in Chaotic Dynamical Systems , it leads to a rich understanding of this emerging field.

This introduction to applied nonlinear dynamics and chaos places emphasis on teaching the techniques and ideas that will enable students to take specific dynamical systems and obtain some quantitative information about their behavior. The new edition has been updated and extended throughout, and contains a detailed glossary of terms. From the reviews: "Will serve as one of the most eminent introductions to the geometric theory of dynamical systems." --Monatshefte für Mathematik

Introduction to Chaos: Physics and Mathematics of Chaotic Phenomena focuses on explaining the fundamentals of the subject by studying examples from one-dimensional maps and simple differential equations. The book includes numerous line diagrams and computer graphics as well as problems and solutions to test readers' understanding. The book i

This book gives a mathematical treatment of the introduction to qualitative differential equations and discrete dynamical systems. The treatment includes theoretical proofs, methods of calculation, and applications. The two parts of the book, continuous time of differential equations and discrete time of dynamical systems, can be covered independently in one semester each or combined together into a year long course. The material on differential equations introduces the qualitative or geometric approach through a treatment of linear systems in any dimension. There follows chapters where equilibria are the most important feature, where scalar (energy) functions is the principal tool, where periodic orbits appear, and finally, chaotic systems of differential equations. The many different approaches are systematically introduced through examples and theorems. The material on discrete dynamical systems starts with maps of one variable and proceeds to systems in higher dimensions. The treatment starts with examples where the periodic points can be found explicitly and then introduces symbolic dynamics to analyze where they can be shown to exist but not given in explicit form. Chaotic systems are presented both mathematically and more computationally using Lyapunov exponents. With the one-dimensional maps as models, the multidimensional maps cover the same material in higher dimensions. This higher dimensional material is less computational and more conceptual and theoretical. The final chapter on fractals introduces various dimensions which is another computational tool for measuring the complexity of a system. It also treats iterated function systems which give examples of complicated sets. In the second edition of the book, much of the material has been rewritten to clarify the presentation. Also, some new material has been included in both parts of the book. This book can be used as a textbook for an advanced undergraduate course on ordinary differential equations and/or dynamical systems. Prerequisites are standard courses in calculus (single variable and multivariable), linear algebra, and introductory differential equations.

Introduction to Mathematical Modeling and Chaotic Dynamics focuses on mathematical models in natural systems, particularly ecological systems. Most of the models presented are solved using MATLAB®. The book first covers the necessary mathematical preliminaries, including testing of stability. It then describes the modeling of systems from natural science, focusing on one- and two-dimensional continuous and discrete time models. Moving on to chaotic dynamics, the authors discuss ways to study chaos, types of chaos, and methods for detecting chaos. They also explore chaotic dynamics in single and multiple species systems. The text concludes with a brief discussion on models of mechanical systems and electronic circuits. Suitable for advanced undergraduate and graduate students, this book provides a practical understanding of how the models are used in current natural science and engineering applications. Along with a variety of exercises and solved examples, the text presents all the fundamental concepts and mathematical skills needed to build models and perform analyses.

The study of dynamical systems is a well established field. This book provides a panorama of several aspects of interest to mathematicians and physicists. It collects the material of several courses at the graduate level given by the authors, avoiding detailed proofs in exchange for numerous illustrations and examples. Apart from common subjects in this field, a lot of attention is given to questions of physical measurement and stochastic properties of chaotic dynamical systems.

The author presents an accessible, clear introduction to dynamical systems and chaos theory, important and exciting areas that have shaped many scientific fields. While the rules governing dynamical systems are well-specified and simple, the behavior of many dynamical systems is remarkably complex.

The materials in the book and on the accompanying disc are not solely developed with only the researcher and professional in mind, but also with consideration for the student: most of this material has been class-tested by the authors. The book is packed with some 100 computer graphics to illustrate the material, and the CD-ROM contains full-colour animations tied directly to the subject matter of the book itself. The cross-platform CD also contains the program ENDO, which enables users to create their own 2-D imagery with X-Windows. Maple scripts are provided to allow readers to work directly with the code from which the graphics in the book were taken.

The book discusses continuous and discrete systems in systematic and sequential approaches for all aspects of nonlinear dynamics. The unique feature of the book is its mathematical theories on flow bifurcations, oscillatory solutions, symmetry analysis of nonlinear systems and chaos theory. The logically structured content and sequential orientation provide readers with a global overview of the topic. A systematic mathematical approach has been adopted, and a number of examples worked out in detail and exercises have been included. Chapters 1–8 are devoted to continuous systems, beginning with one-dimensional flows. Symmetry is an inherent character of nonlinear systems, and the Lie invariance principle and its algorithm for finding symmetries of a system are discussed in Chap. 8. Chapters 9–13 focus on discrete systems, chaos and fractals. Conjugacy relationship among maps and its properties are described with proofs. Chaos theory and its connection with fractals, Hamiltonian flows and symmetries of nonlinear systems are among the main focuses of this book. Over the past few decades, there has been an unprecedented interest and advances in nonlinear systems, chaos theory and fractals, which is reflected in undergraduate and postgraduate curricula around the world. The book is useful for courses in dynamical systems and chaos, nonlinear dynamics, etc., for advanced undergraduate and postgraduate students in mathematics, physics and engineering.

This book presents a new approach for the analysis of chaotic behavior in non-linear dynamical systems, in which output can be represented in quaternion parametrization. It offers a new family of methods for the analysis of chaos in the quaternion domain along with extensive numerical experiments performed on human motion data and artificial data. All methods and algorithms are designed to allow detection of deterministic chaos behavior in quaternion data representing the rotation of a body in 3D space. This book is an excellent reference for engineers, researchers, and postgraduate students conducting research on human gait analysis, healthcare informatics, dynamical systems with deterministic chaos or time series analysis.

For students with a background in elementary algebra, this book provides a vivid introduction to the key phenomena and ideas of chaos and fractals, including the butterfly effect, strange attractors, fractal dimensions, Julia Sets and the Mandelbrot Set, power laws, and cellular automata. The book includes over 200 end-of-chapter exercises.

This text is about the dynamical aspects of ordinary differential equations and the relations between dynamical systems and certain fields outside pure mathematics. It is an update of one of Academic Press's most successful mathematics texts ever published, which has become the standard textbook for graduate courses in this area. The authors are tops in the field of advanced mathematics. Steve Smale is a Field's Medalist, which equates to being a Nobel prize winner in mathematics. Bob Devaney has authored several leading books in this subject area. Linear algebra prerequisites toned down from first edition Inclusion of analysis of examples of chaotic systems, including Lorenz, Rosssler, and Shilnikov systems Bifurcation theory included throughout.

A hundred years ago it became known that deterministic systems can exhibit very complex behavior. By proving that ordinary differential equations can exhibit strange behavior, Poincare undermined the founda tions of Newtonian physics and opened a window to the modern theory of nonlinear dynamics and chaos. Although in the 1930s and 1940s strange behavior was observed in many physical systems, the notion that this phenomenon was inherent in deterministic systems was never suggested. Even with the powerful results of S. Smale in the 1960s, complicated be havior of deterministic systems remained no more than a mathematical curiosity. Not until the late 1970s, with the advent of fast and cheap comput ers, was it recognized that chaotic behavior was prevalent in almost all domains of science and technology. Smale horseshoes began appearing in many scientific fields. In 1971, the phrase 'strange attractor' was coined to describe complicated long-term behavior of deterministic systems, and the term quickly became a paradigm of nonlinear dynamics. The tools needed to study chaotic phenomena are entirely different from those used to study periodic or quasi-periodic systems; these tools are analytic and measure-theoretic rather than geometric. For example, in throwing a die, we can study the limiting behavior of the system by viewing the long-term behavior of individual orbits. This would reveal incomprehensibly complex behavior. Or we can shift our perspective: Instead of viewing the long-term outcomes themselves, we can view the probabilities of these outcomes. This is the measure-theoretic approach taken in this book.

The previous edition of this text was the first to provide a quantitative introduction to chaos and nonlinear dynamics at the undergraduate level. It was widely praised for the clarity of writing and for the unique and effective way in which the authors presented the basic ideas. These same qualities characterize this revised and expanded second edition. Interest in chaotic dynamics has grown explosively in recent years. Applications to practically every scientific field have had a far-reaching impact. As in the first edition, the authors present all the main features of chaotic dynamics using the damped, driven pendulum as the primary model. This second edition includes additional material on the analysis and characterization of chaotic data, and applications of chaos. This new edition of Chaotic Dynamics can be used as a text for courses on chaos for physics and engineering students at the second- and third-year level.

Both fractal geometry and dynamical systems have a long history of development and have provided fertile ground for many great mathematicians and much deep and important mathematics. These two areas interact with each other and with the theory of chaos in a fundamental way: many dynamical systems (even some very simple ones) produce fractal sets, which are in turn a source of irregular 'chaotic' motions in the system. This book is an introduction to these two fields, with an emphasis on the relationship between them. The first half of the book introduces some of the key ideas in fractal geometry and dimension theory - Cantor sets, Hausdorff dimension, box dimension - using dynamical notions whenever possible, particularly one-dimensional Markov maps and symbolic dynamics. Various techniques for computing Hausdorff dimension are shown, leading to a discussion of Bernoulli and Markov measures and of the relationship between dimension, entropy, and Lyapunov exponents. In the second half of the book some examples of dynamical systems are considered and various phenomena of chaotic behaviour are discussed, including bifurcations, hyperbolicity, attractors, horseshoes, and intermittent and persistent chaos. These phenomena are naturally revealed in the course of our study of two real models from science - the FitzHugh - Nagumo model and the Lorenz system of differential equations. This book is accessible to undergraduate students and requires only standard knowledge in calculus, linear algebra, and differential equations. Elements of point set topology and measure theory are introduced as needed. This book is a result of the MASS course in analysis at Penn State University in the fall semester of 2008.

For computer scientists, especially those in the security field, the use of chaos has been limited to the computation of a small collection of famous but unsuitable maps that offer no explanation of why chaos is relevant in the considered contexts. Discrete Dynamical Systems and Chaotic Machines: Theory and Applications shows how to make finite machines, such as computers, neural networks, and wireless sensor networks, work chaotically as defined in a rigorous mathematical framework. Taking into account that these machines must interact in the real world, the authors share their research results on the behaviors of discrete dynamical systems and their use in computer science. Covering both theoretical and practical aspects, the book presents: Key mathematical and physical ideas in chaos theory Computer science fundamentals, clearly establishing that chaos properties can be satisfied by finite state machines Concrete applications of chaotic machines in computer security, including pseudorandom number generators, hash functions, digital watermarking, and steganography Concrete applications of chaotic machines in wireless sensor networks, including secure data aggregation and video surveillance Until the authors’ recent research, the practical implementation of the mathematical theory of chaos on finite machines raised several issues. This self-contained book illustrates how chaos theory enables the study of computer security problems, such as steganalysis, that otherwise could not be tackled. It also explains how the theory reinforces existing cryptographically secure tools and schemes.

Chaos and Nonlinear Dynamics is a comprehensive introduction to the exciting scientific field of nonlinear dynamics for students, scientists, and engineers, and requires only minimal prerequisites in physics and mathematics. The book treats all the important areas in the field and provides an extensive and up-to-date bibliography of applications in all fields of science, social science, economics, and even the arts.

Several distinctive aspects make Dynamical Systems unique, including: treating the subject from a mathematical perspective with the proofs of most of the results included providing a careful review of background materials introducing ideas through examples and at a level accessible to a beginning graduate student

Introduces the mathematical topics of chaos, fractals, and dynamics using a combination of hands-on computer experimentation and precalculas mathmetics. A series of experiments produce fascinating computer graphics images of Julia sets, the Mandelbrot set, and fractals. The basic ideas of dynamics--chaos, iteration, and stability--are illustrated via computer projects.

This text discusses the qualitative properties of dynamical systems including both differential equations and maps. The approach taken relies heavily on examples (supported by extensive exercises, hints to solutions and diagrams) to develop the material, including a treatment of chaotic behavior. The unprecedented popular interest shown in recent years in the chaotic behavior of discrete dynamic systems including such topics as chaos and fractals has had its impact on the undergraduate and graduate curriculum. However there has, until now, been no text which sets out this developing area of mathematics within the context of standard teaching of ordinary differential equations. Applications in physics, engineering, and geology are considered and introductions to fractal imaging and cellular automata are given.

Over the past two decades scientists, mathematicians, and engineers have come to understand that a large variety of systems exhibit complicated evolution with time. This complicated behavior is known as chaos. In the new edition of this classic textbook Edward Ott has added much new material and has significantly increased the number of homework problems. The most important change is the addition of a completely new chapter on control and synchronization of chaos. Other changes include new material on riddled basins of attraction, phase locking of globally coupled oscillators, fractal aspects of fluid advection by Lagrangian chaotic flows, magnetic dynamos, and strange nonchaotic attractors. This new edition will be of interest to advanced undergraduates and graduate students in science, engineering, and mathematics taking courses in chaotic dynamics, as well as to researchers in the subject.

An Introduction To Chaotic Dynamical SystemsCRC Press

BACKGROUND Sir Isaac Newton hrought to the world the idea of modeling the motion of physical systems with equations. It was necessary to invent calculus along the way, since fundamental equations of motion involve velocities and accelerations, of position. His greatest single success was his discovery that which are derivatives the motion of the planets and moons of the solar system resulted from a single fundamental source: the gravitational attraction of the hodies. He demonstrated that the ohserved motion of the planets could he explained hy assuming that there is a gravitational attraction he tween any two ohjects, a force that is proportional to the product of masses and inversely proportional to the square of the distance between them. The circular, elliptical, and parabolic orhits of astronomy were v INTRODUCTION no longer fundamental determinants of motion, but were approximations of laws specified with differential equations. His methods are now used in modeling motion and change in all areas of science. Subsequent generations of scientists extended the method of using differ ential equations to describe how physical systems evolve. But the method had a limitation. While the differential equations were sufficient to determine the behavior-in the sense that solutions of the equations did exist-it was frequently difficult to figure out what that behavior would be. It was often impossible to write down solutions in relatively simple algebraic expressions using a finite number of terms. Series solutions involving infinite sums often would not converge beyond some finite time.