Introduction To Geometric Measure Theory And The Plateau

This contributed volume collects papers based on courses and talks given at the 2017 CIMPA school Harmonic Analysis, Geometric Measure Theory and Applications, which took place at the University of Buenos Aires in August 2017. These articles highlight recent breakthroughs in both harmonic analysis and geometric measure theory, particularly focusing on their impact on image and signal processing. The wide range of expertise present in these articles will help readers contextualize how these breakthroughs have been instrumental in resolving deep theoretical problems. Some topics covered include: Gabor frames Falconer distance problem Hausdorff dimension Sparse inequalities Fractional Brownian motion Fourier analysis in geometric measure theory This volume is ideal for applied and pure mathematicians interested in the areas of image and signal processing. Electrical engineers and statisticians studying these fields will also find this to be a valuable resource.

Modern text examining the interplay between measure theory and Fourier analysis.

This compact and well-received book, now in its second edition, is a skilful combination of measure theory and probability. For, in contrast to many books where probability theory is usually developed after a thorough exposure to the theory and techniques of measure and integration, this text develops the Lebesgue theory of measure and integration, using probability theory as the motivating force. What distinguishes the text is the illustration of all theorems by examples and applications. A section on Stieltjes integration assists the student in understanding the later text better. For easy understanding and presentation, this edition has split some long chapters into smaller ones. For example, old Chapter 3 has been split into Chapters 3 and 9, and old Chapter 11 has been split into Chapters 11, 12 and 13. The book is intended for the first-year postgraduate students for their courses in Statistics and Mathematics (pure and applied), computer science, and electrical and industrial engineering. KEY FEATURES : Measure theory and probability are well integrated. Exercises are given at the end of each chapter, with solutions provided separately. A section is devoted to large sample theory of statistics, and another to large deviation theory (in the Appendix).

This book collects together lectures by some of the leaders in the field of partial differential equations and geometric measure theory. It features a wide variety of research topics in which a crucial role is played by the interaction of fine analytic techniques and deep geometric observations, combining the intuitive and geometric aspects of mathematics with analytical ideas and variational methods. The problems addressed are challenging and complex, and often require the use of several refined techniques to overcome the major difficulties encountered. The lectures, given during the course "Partial Differential Equations and Geometric Measure Theory" in Cetraro, June 2–7, 2014, should help to encourage further research in the area. The enthusiasm of the speakers and the participants of this CIME course is reflected in the text.

The book provides a comprehensive introduction and a novel mathematical foundation of the field of information geometry with complete proofs and detailed background material on measure theory, Riemannian geometry and Banach space theory. Parametrised measure models are defined as fundamental geometric objects, which can be both finite or infinite dimensional. Based on these models, canonical tensor fields are introduced and further studied, including the Fisher metric and the Amari-Chentsov tensor, and embeddings of statistical manifolds are investigated. This novel foundation then leads to application highlights, such as generalizations and extensions of the classical uniqueness result of Chentsov or the Cramér-Rao inequality. Additionally, several new application fields of information geometry are highlighted, for instance hierarchical and graphical models, complexity theory, population genetics, or Markov Chain Monte Carlo. The book will be of interest to mathematicians who are interested in geometry, information theory, or the foundations of statistics, to statisticians as well as to scientists interested in the mathematical foundations of complex systems.

This is a graduate text introducing the fundamentals of measure theory and integration theory, which is the foundation of modern real analysis. The text focuses first on the concrete setting of Lebesgue measure and the Lebesgue integral (which in turn is motivated by the more classical concepts of Jordan measure and the Riemann integral), before moving on to abstract measure and integration theory, including the standard convergence theorems, Fubini's theorem, and the Carathéodory extension theorem. Classical differentiation theorems, such as the Lebesgue and Rademacher differentiation theorems, are also covered, as are connections with probability theory. The material is intended to cover a quarter or semester's worth of material for a first graduate course in real analysis. There is an emphasis in the text on tying together the abstract and the concrete sides of the subject, using the latter to illustrate and motivate the former. The central role of key principles (such as Littlewood's three principles) as providing guiding intuition to the subject is also emphasized. There are a large number of exercises throughout that develop key aspects of the theory, and are thus an integral component of the text. As a supplementary section, a discussion of general problem-solving strategies in analysis is also given. The last three sections discuss optional topics related to the main matter of the book.

In 2013, a school on Geometric Measure Theory and Real Analysis, organized by G. Alberti, C. De Lellis and myself, took place at the Centro De Giorgi in Pisa, with lectures by V. Bogachev, R. Monti, E. Spadaro and D. Vittone. The book collects the notes of the courses. The courses provide a deep and up to date insight on challenging mathematical problems and their recent developments: infinite-dimensional analysis, minimal surfaces and isoperimetric problems in the Heisenberg group, regularity of sub-Riemannian geodesics and the regularity theory of minimal currents in any dimension and codimension.

A one-stop introduction to the methods of ergodic theory applied to holomorphic iteration that is ideal for graduate courses.

These twenty-six papers survey a cross section of current work in modern geometric measure theory and its applications in the calculus of variations. Presently the field consists of a jumble of new ideas, techniques and intuitive hunches; an exchange of information has been hindered, however, by the characteristic length and complexity of formal research papers in higher-dimensional geometric analysis. This volume provides an easier access to the material, including introductions and summaries of many of the authors' much longer works and a section containing 80 open problems in the field. The papers are aimed at analysts and geometers who may use geometric measure-theoretic techniques, and they require a mathematical sophistication at the level of a second year graduate student. The papers included were presented at the 1984 AMS Summer Research Institute held at Humboldt State University. A major theme of this institute was the introduction and application of multiple-valued function techniques as a basic new tool in geometric analysis, highlighted by Almgren's fundamental paper Deformations and multiple-valued functions. Major new results discussed at the conference included the following: Allard's integrality and regularity theorems for surfaces stationary with respect to general elliptic integrands; Scheffer's first example of a singular solution to the Navier-Stokes equations for a fluid flow with opposing force; and Hutchinson's new definition of the second fundamental form of a general varifold.

This work is intended to give a quick overview on the subject of the geometric measure theory with emphases on various basic ideas, techniques and their applications in problems arising in the calculus of variations, geometrical analysis and nonlinear partial differential equations.

In this book we introduce the class of mappings of finite distortion as a generalization of the class of mappings of bounded distortion. Connections with models of nonlinear elasticity are also discussed. We study continuity properties, behavior of our mappings on null sets, topological properties like openness and discreteness, regularity of the potential inverse mappings and many other aspects.

Geometric Measure Theory: A Beginner's Guide provides information pertinent to the development of geometric measure theory. This book presents a few fundamental arguments and a superficial discussion of the regularity theory. Organized into 12 chapters, this book begins with an overview of the purpose and fundamental concepts of geometric measure theory. This text then provides the measure-theoretic foundation, including the definition of Hausdorff measure and covering theory. Other chapters consider the m-dimensional surfaces of geometric measure theory called rectifiable sets and introduce the two basic tools of the regularity theory of area-minimizing surfaces. This book discusses as well the fundamental theorem of geometric measure theory, which guarantees solutions to a wide class of variational problems in general dimensions. The final chapter deals with the basic methods of geometry and analysis in a generality that embraces manifold applications. This book is a valuable resource for graduate students, mathematicians, and research workers.

This textbook introduces geometric measure theory through the notion of currents. Currents, continuous linear functionals on spaces of differential forms, are a natural language in which to formulate types of extremal problems arising in geometry, and can be used to study generalized versions of the Plateau problem and related questions in geometric analysis. Motivating key ideas with examples and figures, this book is a comprehensive introduction ideal for both self-study and for use in the classroom. The exposition demands minimal background, is self-contained and accessible, and thus is ideal for both graduate students and researchers.

This textbook collects the notes for an introductory course in measure theory and integration. The course was taught by the authors to undergraduate students of the Scuola Normale Superiore, in the years 2000-2011. The goal of the course was to present, in a quick but rigorous way, the modern point of view on measure theory and integration, putting Lebesgue's Euclidean space theory into a more general context and presenting the basic applications to Fourier series, calculus and real analysis. The text can also pave the way to more advanced courses in probability, stochastic processes or geometric measure theory. Prerequisites for the book are a basic knowledge of calculus in one and several variables, metric spaces and linear algebra. All results presented here, as well as their proofs, are classical. The authors claim some originality only in the presentation and in the choice of the exercises. Detailed solutions to the exercises are provided in the final part of the book.

This contributed volume explores the applications of various topics in modern differential geometry to the foundations of continuum mechanics. In particular, the contributors use notions from areas such as global analysis, algebraic topology, and geometric measure theory. Chapter authors are experts in their respective areas, and provide important insights from the most recent research. Organized into two parts, the book first covers kinematics, forces, and stress theory, and then addresses defects, uniformity, and homogeneity. Specific topics covered include: Global stress and hyper-stress theories Applications of de Rham currents to singular dislocations Manifolds of mappings for continuum mechanics Kinematics of defects in solid crystals Geometric Continuum Mechanics will appeal to graduate students and researchers in the fields of mechanics, physics, and engineering who seek a more rigorous mathematical understanding of the area. Mathematicians interested in applications of analysis and geometry will also find the topics covered here of interest.

This open access textbook welcomes students into the fundamental theory of measure, integration, and real analysis. Focusing on an accessible approach, Axler lays the foundations for further study by promoting a deep understanding of key results. Content is carefully curated to suit a single course, or two-semester sequence of courses, creating a versatile entry point for graduate studies in all areas of pure and applied mathematics. Motivated by a brief review of Riemann integration and its deficiencies, the text begins by immersing students in the concepts of measure and integration. Lebesgue measure and abstract measures are developed together, with each providing key insight into the main ideas of the other approach. Lebesgue integration links into results such as the Lebesgue Differentiation Theorem. The development of products of abstract measures leads to Lebesgue measure on Rn. Chapters on Banach spaces, Lp spaces, and Hilbert spaces showcase major results such as the Hahn–Banach Theorem, Hölder's Inequality, and the Riesz Representation Theorem. An in-depth study of linear maps on Hilbert spaces culminates in the Spectral Theorem and Singular Value Decomposition for compact operators, with an optional interlude in real and complex measures. Building on the Hilbert space material, a chapter on Fourier analysis provides an invaluable introduction to Fourier series and the Fourier transform. The final chapter offers a taste of probability. Extensively class tested at multiple universities and written by an award-winning mathematical expositor, Measure, Integration & Real Analysis is an ideal resource for students at the start of their journey into graduate mathematics. A prerequisite of elementary undergraduate real analysis is assumed; students and instructors looking to reinforce these ideas will appreciate the electronic Supplement for Measure, Integration & Real Analysis that is freely available online.

This book gives an up-to-date account of progress on Pansu's celebrated problem on the sub-Riemannian isoperimetric profile of the Heisenberg group. It also serves as an introduction to the general field of sub-Riemannian geometric analysis. It develops the methods and tools of sub-Riemannian differential geometry, nonsmooth analysis, and geometric measure theory suitable for attacks on Pansu's problem.

When it was first published this was the first general account of Hausdorff measures, a subject that has important applications in many fields of mathematics. There are three chapters: the first contains an introduction to measure theory, paying particular attention to the study of non-s-finite measures. The second develops the most general aspects of the theory of Hausdorff measures, and the third gives a general survey of applications of Hausdorff measures followed by detailed accounts of two special applications. This edition has a foreword by Kenneth Falconer outlining the developments in measure theory since this book first appeared. Based on lectures given by the author at University College London, this book is ideal for graduate mathematicians with no previous knowledge of the subject, but experts in the field will also want a copy for their shelves.

A mathematical study of the geometrical aspects of sets of both integral and fractional Hausdorff dimension. Considers questions of local density, the existence of tangents of such sets as well as the dimensional properties of their projections in various directions.

This book studies the geometric properties of general sets and measures in euclidean space.
This volume covers contemporary aspects of geometric measure theory with a focus on applications to partial differential equations, free boundary problems and water waves. It is based on lectures given at the 2019 CIME summer school “Geometric Measure Theory and Applications – From Geometric Analysis to Free Boundary Problems” which took place in Cetraro, Italy, under the scientific direction of Matteo Focardi and Emanuele Spadaro. Providing a description of the structure of measures satisfying certain differential constraints, and covering regularity theory for Bernoulli type free boundary problems and water waves as well as regularity theory for the obstacle problems and the developments leading to applications to the Stefan problem, this volume will be of interest to students and researchers in mathematical analysis and its applications.
W.K. ALLARD: On the first variation of area and generalized mean curvature.- F.J. ALMGREN Jr.: Geometric measure theory and elliptic variational problems.- E. GIUSTI: Minimal surfaces with obstacles.- J. GUCKENHEIMER: Singularities in soap-bubble-like and soap-film-like surfaces.- D. KINDERLEHRER: The analyticity of the coincidence set in variational inequalities.- M. MIRANDA: Boundaries of Caciopoli sets in the calculus of variations.- L. PICCININI: De Giorgi’s measure and thin obstacles.
This book explains the notion of Brakke’s mean curvature flow and its existence and regularity theories without assuming familiarity with geometric measure theory. The focus of study is a time-parameterized family of k-dimensional surfaces in the n-dimensional Euclidean space (1 ? k in
"This book is a major treatise in mathematics and is essential in the working library of the modern analyst." (Bulletin of the London Mathematical Society)
This book is based on lecture notes for a short course for Masters level or senior undergraduate students. It may also serve as easy (and hopefully pleasant) reading for researchers in a different field of Mathematics. The main purpose of the book is to look closely at some results that are basic for modern Analysis and which fascinated the author when she was a student, and to show how they constitute a foundation for the branch of Analysis known as Geometric Measure Theory. The secondary aim of the book is to give a straightforward but reasonably complete introduction to the definition of Hausdorff measure and Hausdorff dimension and to illustrate how non-trivial they can be. The course has no ambition to replace a serious course on Geometric Measure Theory, but rather to encourage the student to take such a course. The author comes from Russia. For the past 17 years she has worked at Chalmers University of Technology in Gothenburg, Sweden. She also had visiting positions in Canada, France, and Poland.
This self-contained treatment of measure and integration begins with a brief review of the Riemann integral and proceeds to a construction of Lebesgue measure on the real line. From there the reader is led to the general notion of measure, to the construction of the Lebesgue integral on a measure space, and to the major limit theorems, such as the Monotone and Dominated Convergence Theorems. The treatment proceeds to L^p spaces, normed linear spaces that are shown to be complete (i.e., Banach spaces) due to the limit theorems. Particular attention is paid to L^2 spaces as Hilbert spaces, with a useful geometrical structure. Having gotten quickly to the heart of the matter, the text proceeds to broaden its scope. There are further constructions of measures, including Lebesgue measure on n-dimensional Euclidean space. There are also discussions of surface measure, and more generally of Riemannian manifolds and the measures they inherit, and an appendix on the integration of differential forms. Further geometric aspects are explored in a chapter on Hausdorff measure. The text also treats probabilistic concepts, in chapters on ergodic theory, probability spaces and random variables, Wiener measure and Brownian motion, and martingales. This text will prepare graduate students for more advanced studies in functional analysis, harmonic analysis, stochastic analysis, and geometric measure theory.
This volume develops the classical theory of the Lebesgue integral and some of its applications. The integral is initially presented in the context of n-dimensional Euclidean space, following a thorough study of the concepts of outer measure and measure. A more general treatment of the integral, based on an axiomatic approach, is later given. Closely related topics in real variables, such as functions of bounded variation, the Riemann-Stieltjes integral, Fubini's theorem, L(p)) classes, and various results about differentiation are examined in detail. Several applications of the theory to a specific branch of analysis--harmonic analysis--are also provided. Among these applications are basic facts about convolution operators and Fourier series, including results for the conjugate function and the Hardy-Littlewood maximal function.
Measure and Integral: An Introduction to Real Analysis provides an introduction to real analysis for student interested in mathematics, statistics, or probability. Requiring only a basic familiarity with advanced calculus, this volume is an excellent textbook for advanced undergraduate or first-year graduate student in these areas.
Inspired by classical geometry, geometric group theory has in turn provided a variety of applications to geometry, topology, group theory, number theory and graph theory. This carefully written textbook provides a rigorous introduction to this rapidly evolving field whose methods have proven to be powerful tools in neighbouring fields such as geometric topology. Geometric group theory is the study of finitely generated groups via the geometry of their associated Cayley graphs. It turns out that the essence of the geometry of such groups is captured in the key notion of quasi-isometry, a large-scale version of isometry whose invariants include growth types, curvature conditions, boundary constructions, and amenability. This book covers the foundations of quasi-geometry of groups at an advanced undergraduate level. The subject is illustrated by many elementary examples, outlooks on applications, as well as an extensive collection of exercises.
The characterization of rectifiable sets through the existence of densities is a pearl of geometric measure theory. The difficult proof, due to Preiss, relies on many beautiful and deep ideas and novel techniques. Some of them have already proven useful in other contexts, whereas others have not yet been exploited. These notes give a simple and short presentation of the former and provide some perspective of the latter. This text emerged from a course on rectifiability

given at the University of Zurich. It is addressed both to researchers and students; the only prerequisite is a solid knowledge in standard measure theory. The first four chapters give an introduction to rectifiable sets and measures in Euclidean spaces, covering classical topics such as the area formula, the theorem of Marstrand and the most elementary rectifiability criterions. The fifth chapter is dedicated to a subtle rectifiability criterion due to Marstrand and generalized by Mattila, and the last three focus on Preiss' result. The aim is to provide a self-contained reference for anyone interested in an overview of this fascinating topic.

The basic ideas of the subject and the analogues with enumerative combinatorics are described and exploited.

An engaging graduate-level introduction that bridges analysis and geometry. Suitable for self-study and a useful reference for researchers.

Measure Theory and Fine Properties of Functions, Revised Edition provides a detailed examination of the central assertions of measure theory in n-dimensional Euclidean space. The book emphasizes the roles of Hausdorff measure and capacity in characterizing the fine properties of sets and functions. Topics covered include a quick review of abstract

Geared toward upper-level undergraduates and graduate students, this treatment of geometric integration theory consists of an introduction to classical theory, a postulational approach to general theory, and a section on Lebesgue theory. 1957 edition.

Geometric Measure Theory: A Beginner's Guide, Fifth Edition provides the framework readers need to understand the structure of a crystal, a soap bubble cluster, or a universe. The book is essential to any student who wants to learn geometric measure theory, and will appeal to researchers and mathematicians working in the field. Brevity, clarity, and scope make this classic book an excellent introduction to more complex ideas from geometric measure theory and the calculus of variations for beginning graduate students and researchers. Morgan emphasizes geometry over proofs and technicalities, providing a fast and efficient insight into many aspects of the subject, with new coverage to this edition including topical coverage of the Log Convex Density Conjecture, a major new theorem at the center of an area of mathematics that has exploded since its appearance in Perelman's proof of the Poincaré conjecture, and new topical coverage of manifolds taking into account all recent research advances in theory and applications. Focuses on core geometry rather than proofs, paving the way to fast and efficient insight into an extremely complex topic in geometric structures Enables further study of more advanced topics and texts Demonstrates in the simplest possible way how to relate concepts of geometric analysis by way of algebraic or topological techniques Contains full topical coverage of The Log-Convex Density Conjecture Comprehensively updated throughout

This book introduces a new research direction in set theory: the study of models of set theory with respect to their extensional overlap or disagreement. In Part I, the method is applied to isolate new distinctions between Borel equivalence relations. Part II contains applications to independence results in Zermelo–Fraenkel set theory without Axiom of Choice. The method makes it possible to classify in great detail various paradoxical objects obtained using the Axiom of Choice; the classifying criterion is a ZF-provable implication between the existence of such objects. The book considers a broad spectrum of objects from analysis, algebra, and combinatorics: ultrafilters, Hamel bases, transcendence bases, colorings of Borel graphs, discontinuous homomorphisms between Polish groups, and many more. The topic is nearly inexhaustible in its variety, and many directions invite further investigation.