Lecture Notes On C Algebras And K Theory

The notion of a motive is an elusive one, like its namesake "the motif" of Cezanne's impressionist method of painting. Its existence was first suggested by Grothendieck in 1964 as the underlying structure behind the myriad cohomology theories in Algebraic Geometry. We now know that there is a triangulated theory of motives, discovered by Vladimir Voevodsky, which suffices for the development of a satisfactory Motivic Cohomology theory. However, the existence of motives themselves remains conjectural. This book provides an account of the triangulated theory of motives. Its purpose is to introduce Motivic Cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, etale cohomology, and Chow groups. The book is divided into lectures, grouped in six parts. The first part presents the definition of Motivic Cohomology, based upon the notion of presheaves with transfers. Some elementary comparison theorems are given in this part. The theory of (etale, Nisnevich, and Zariski) sheaves with transfers is developed in parts two, three, and six, respectively. The theoretical core of the book is the fourth part, presenting the triangulated category of motives. Finally, the comparison with higher Chow groups is developed in part five. The lecture notes format is designed for the book to be read by an advanced graduate student or an expert in a related field. The lectures roughly correspond to one-hour lectures given by Voevodsky during the course he gave at the Institute for Advanced Study in Princeton on this subject in 1999-2000. In addition, many of the original proofs have been simplified and improved so that this book will also be a useful tool for research mathematicians. Information for our distributors: Titles in this series are copublished with the Clay Mathematics Institute (Cambridge, MA).

This volume presents the lecture notes of short courses given by three leading experts in mathematical logic at the 2012 Asian Initiative for Infinity Logic Summer School. The major topics cover set-theoretic forcing, higher recursion theory, and applications of set theory to C*-algebra. This volume offers a wide spectrum of ideas and techniques introduced in contemporary research in the field of mathematical logic to students, researchers and mathematicians.

Noncommutative Geometry is one of the most deep and vital research subjects of present-day Mathematics. Its development, mainly due to Alain Connes, is providing an increasing number of applications and deeper insights for instance in Foliations, K-Theory, Index Theory, Number Theory but also in Quantum Physics of elementary particles. The purpose of the Summer School in Martina Franca was to offer a fresh invitation to the subject and closely related topics; the contributions in this volume include the four main lectures, cover advanced developments and are delivered by prominent specialists.

This book contains a collection of articles provided by the participants of the SFB-workshop on C*-algebras, March 8 - March 12, 1999 which was held at the Sonderforschungsbereich "Geometrische Strukturen in der reinen Mathematik" of the University of Münster, Germany. The aim of the workshop was to bring together leading experts in the theory of C* -algebras with promising young researchers in the field, and to provide a stimulating atmosphere for discussions and interactions between the participants. There were 19 one-hour lectures on various topics like - classification of nuclear C* -algebras, - general K-theory for C* -algebras, - exact C* -algebras and exact groups, - C*-algebras associated to (infinite) matrices and C*-correspondences, - noncommutative prob ability theory, - deformation quantization, - group C* -algebras and the Baum-Connes conjecture, giving a broad overview of the latest developments in the field, and serving as a basis for discussions. We, the organizers of the workshop, were greatly pleased with the excellence of the lectures and so were led to the idea of publishing the proceedings of the conference. There are basically two kinds of contributions. On one side there are several articles giving surveys and overviews on new developments and im portant results of the theory, on the other side one finds original articles with interesting new results.

This book is directed towards graduate students that wish to start from the basic theory of C*-algebras and advance to an overview of some of the most spectacular results concerning the structure of nuclear C*-algebras. The text is divided into three parts. First, elementary notions, classical theorems and constructions are developed. Then, essential examples in the theory, such as crossed products and the class of quasidiagonal C*-algebras, are examined, and finally, the Elliott invariant, the Cuntz semigroup, and the Jiang-Su algebra are defined. It is shown how these objects have played a fundamental role in understanding the fine structure of nuclear C*-algebras. To help understanding the theory, plenty of examples, treated in detail, are included. This volume will also be valuable to researchers in the area as a reference guide. It contains an extensive reference list to guide readers that wish to travel further.

Based on the author's university lecture courses, this book presents the many facets of one of the most important open problems in operator algebra theory. Central to this book is the proof of the equivalence of the various forms of the problem, including forms involving C*-algebra tensor products and free groups, ultraproducts of von Neumann algebras, and quantum information theory. The reader is guided through a number of results (some of them previously unpublished) revolving around tensor products of C*-algebras and operator spaces, which are reminiscent of Grothendieck's famous Banach space theory work. The detailed style of the book and the inclusion of background information make it easily accessible for beginning researchers, Ph.D. students, and non-specialists alike.

The "extensions" of rings and modules have yet to be explored in detail in a research monograph. This book presents state of the art research and also stimulating new and further research. Broken into three parts, Part I begins with basic notions, terminology, definitions and a description of the classes of rings and modules. Part II considers the transference of conditions between a base ring or module and its extensions. And Part III utilizes the concept of a minimal essental extension with respect to a specific class (a hull). Mathematical interdisciplinary applications appear throughout. Major applications of the ring and module theory to Functional Analysis, especially C*-algebras, appear in Part III, make this book of interest to Algebra and Functional Analysis researchers. Notes and exercises at the end of every chapter, and open problems at the end of all three parts, lend this as an ideal textbook for graduate or advanced undergradate students.

The amount of algebraic topology a graduate student specializing in topology must learn can be intimidating. Moreover, by their second year of graduate studies, students must make the transition from understanding simple proofs line-by-line to understanding the overall structure of proofs of difficult theorems. To help students make this transition, the material in this book is presented in an increasingly sophisticated manner. It is intended to bridge the gap between algebraic and geometric topology, both by providing the algebraic tools that a geometric topologist needs and by concentrating on those areas of algebraic topology that are geometrically motivated. Prerequisites for using this book include basic set-theoretic topology, the definition of CW-complexes, some knowledge of the fundamental group/covering space theory, and the construction of singular homology. Most of this material is briefly reviewed at the beginning of the book. The topics discussed by the authors include typical material for first- and second-year graduate courses. The core of the exposition consists of chapters on homotopy groups and on spectral sequences. There is also material that would interest students of geometric topology (homology with local coefficients and obstruction theory) and algebraic topology (spectra and generalized homology), as well as preparation for more advanced topics such as algebraic K-theory and the s-cobordism theorem. A unique feature of the book is the inclusion, at the end of each chapter, of several projects that require students to present proofs of substantial theorems and to write notes accompanying their explanations. Working on these projects allows students to grapple with the ``big picture'', teaches them how to give mathematical lectures, and prepares them for participating in research seminars. The book is designed as a textbook for graduate students studying algebraic and geometric topology and homotopy theory. It will also be useful for students from other fields such as differential geometry, algebraic geometry, and homological algebra. The exposition in the text is clear; special cases are presented over complex general statements.

The 2-volume book is an updated, reorganized and considerably enlarged version of the previous edition of the Research Problem Book in Analysis (LNM 1043), a collection familiar to many analysts, that has sparked off much research. This new edition, created in a joint effort by a large team of analysts, is, like its predecessor, a collection of unsolved problems of modern analysis designed as informally written mini-articles, each containing not only a statement of a problem but also historical and methodological comments, motivation, conjectures and discussion of possible connections, of plausible approaches as well as a list of references. There are now 342 of these mini- articles, almost twice as many as in the previous edition, despite the fact that a good deal of them have been solved!

This book presents the basic tools of modern analysis within the context of the fundamental problem of operator theory: to calculate spectra of specific operators on infinite dimensional spaces, especially operators on Hilbert spaces. The tools are diverse, and they provide the basis for more refined methods that allow one to approach problems that go well beyond the computation of spectra: the mathematical foundations of quantum physics, noncommutative K-theory, and the classification of simple C*-algebras being three areas of current research activity which require mastery of the material presented here.

This book constitutes a first- or second-year graduate course in operator theory. It is a field that has great importance for other areas of mathematics and physics, such as algebraic topology, differential geometry, and quantum mechanics. It assumes a basic knowledge in functional analysis but no prior acquaintance with operator theory is required.

This research monograph introduces some new aspects to the theory of harmonic functions and related topics. The authors study the analytic algebraic structures of the space of bounded harmonic functions on locally compact groups and its non-commutative analogue, the space of harmonic functionals on Fourier algebras. Both spaces are shown to be the range of a contractive projection on a von Neumann algebra and therefore admit Jordan algebraic structures. This provides a natural setting to apply recent results from non-associative analysis, semigroups and Fourier algebras. Topics discussed include Poisson representations, Poisson spaces, quotients of Fourier algebras and the Murray-von Neumann classification of harmonic functionals.

The subject of C*-algebras received a dramatic revitalization in the 1970s by the introduction of topological methods through the work of Brown, Douglas, and Fillmore on extensions of C*-algebras and Elliott's use of K-theory to provide a useful classification of AF algebras. These results were the beginning of a marvelous new set of tools for analyzing concrete C*-algebras. This book is an introductory graduate level text which presents the basics of the subject through a detailed analysis of several important classes of C*-algebras. The development of operator algebras in the last twenty years has been based on a careful study of these special classes. While there are many books on C*-algebras and operator algebras available, this is the first one to attempt to explain the real examples that researchers use to test their hypotheses. Topic include AF algebras, Bunce-Deddens and Cuntz algebras, the Toeplitz algebra, irrational rotation algebras, group C*-algebras, discrete crossed products, abelian C*-algebras (spectral theory and approximate unitary equivalence) and extensions. It also introduces many modern concepts and results in the subject such as real rank zero algebras, topological stable rank, quasidiagonality, and various new constructions. These notes were compiled during the author's participation in the special year on C*-algebras at the Fields Institute of Mathematics during the 1994-1995 academic year. The field of C*-algebras touches upon many other areas of mathematics such as group representations, dynamical systems, physics, K-theory, and topology. The variety of examples offered in this text expose the student to many of these connections. A graduate student with a solid course in functional analysis should be able to read this book. This should prepare them to read much of the current literature. This book is reasonably self-contained, and the author has provided results from other areas when necessary.

K -Theory has revolutionized the study of operator algebras in the last few years. As the primary component of the subject of "noncommutative topol ogy," K -theory has opened vast new vistas within the structure theory of C* algebras, as well as leading to profound and unexpected applications of opera tor algebras to problems in geometry and topology. As a result, many topolo gists and operator algebraists have feverishly begun trying to learn each others' subjects, and it appears certain that these two branches of mathematics have become deeply and permanently intertwined. Despite the fact that the whole subject is only about a decade old, operator K -theory has now reached a state of relative stability. While there will undoubtedly be many more revolutionary developments and applications in the future, it appears the basic theory has more or less reached a "final form." But because of the newness of the theory, there has so far been no comprehensive treatment of the subject. It is the ambitious goal of these notes to fill this gap. We will develop the K -theory of Banach algebras, the theory of extensions of C*-algebras, and the operator K -theory of Kasparov from scratch to its most advanced aspects. We will not treat applications in detail; however, we will outline the most striking of the applications to date in a section at the end, as well as mentioning others at suitable points in the text.

Handbook of Algebra

In this book, we study, under the name of von Neumann algebras, those algebras generally known as “rings of operators“ or “W*-algebras.“ The new terminology, suggested by J. Dieudonng, is fully justified from the historical point of view. Certain of the results are valid for more general algebras. We have, however systematically avoided this kind of generalization, except when it would facilitate the study of von Neumann algebras themselves. Parts I and I1 comprise those results which at present appear to’be the most useful for applications, although we do not embark on the study of those applications. Part 111, which is more technical, is primarily intended for specialists; it is virtually independent of Part 11.

The notion of amenability has its origins in the beginnings of modern measure theory: Does a finitely additive set function exist which is invariant under a certain group action? Since the 1940s, amenability has become an important concept in abstract harmonic analysis (or rather, more generally, in the theory of semitopological semigroups). In 1972, B.E. Johnson showed that the amenability of a locally compact group G can be characterized in terms of the Hochschild cohomology of its group algebra L^1(G): this initiated the theory of amenable Banach algebras. Since then, amenability has penetrated other branches of mathematics, such as von Neumann algebras, operator spaces, and even differential geometry. Lectures on Amenability introduces second year graduate students to this fascinating area of modern mathematics and leads them to a level from where they can go on to read original papers on the subject. Numerous exercises are interspersed in the text.

Cluster algebras are combinatorially defined commutative algebras which were introduced by S. Fomin and A. Zelevinsky as a tool for studying the dual canonical basis of a quantized enveloping algebra and totally positive matrices. The aim of these notes is to give an introduction to cluster algebras which is accessible to graduate students or researchers interested in learning more about the field while giving a taste of the wide connections between cluster algebras and other areas of mathematics. The approach taken emphasizes combinatorial and geometric aspects of cluster algebras. Cluster algebras of finite type are classified by the Dynkin diagrams, so a short introduction to reflection groups is given in order to describe this and the corresponding generalized associahedra. A discussion of cluster algebra periodicity, which has a close relationship with discrete integrable systems, is included. This book ends with a description of the cluster algebras of finite mutation type and the cluster structure of the homogeneous coordinate ring of the Grassmannian, both of which have a beautiful description in terms of combinatorial geometry.

This book is an introductory text on one of the most important fields of Mathematics, the theory of operator algebras. It offers a

readable exposition of the basic concepts, techniques, structures and important results of operator algebras. Written in a self-contained manner, with an emphasis on understanding, it serves as an ideal text for graduate students.
This volume contains the proceedings of an AMS Special Session held at the Joint Mathematics Meetings in San Antonio in January 1993 to celebrate the first fifty years of C^*-algebra theory. The book contains carefully written expository and research articles by leaders in the field. Also included is a reprinting of the original 1943 paper on C^*-algebras by Gelfand and Neumark, which has had such a profound influence on the field. The volume covers a broad spectrum of topics, including the Gelfand-Neumark theorems, C^*-algebras and quantization, projections in C^*-algebras, Mackey's theory of group representations and their relation to C^*-algebras, transformation group C^*-algebras, the influence of algebraic topology on C^*-algebras, K-theory and index theory in operator algebras, exponential rank in C^*-algebras, and a survey of the development of type III von Neumann algebras. With historical perspectives and up-to-date overviews to orient readers new to the field, this book will interest mathematicians, physicists, and mathematical historians.
These lecture notes are an introduction to several ideas and applications of noncommutative geometry. It starts with a not necessarily commutative but associative algebra which is thought of as the algebra of functions on some 'virtual noncommutative space'. Attention is switched from spaces, which in general do not even exist, to algebras of functions. In these notes, particular emphasis is put on seeing noncommutative spaces as concrete spaces, namely as a collection of points with a topology. The necessary mathematical tools are presented in a systematic and accessible way and include among other things, C'*-algebras, module theory and K-theory, spectral calculus, forms and connection theory. Application to Yang--Mills, fermionic, and gravity models are described. Also the spectral action and the related invariance under automorphism of the algebra is illustrated. Some recent work on noncommutative lattices is presented. These lattices arose as topologically nontrivial approximations to 'contuinuum' topological spaces. They have been used to construct quantum-mechanical and field-theory models, alternative models to lattice gauge theory, with nontrivial topological content. This book will be essential to physicists and mathematicians with an interest in noncommutative geometry and its uses in physics.
The theory of crossed products is extremely rich and intriguing. There are applications not only to operator algebras, but to subjects as varied as noncommutative geometry and mathematical physics. This book provides a detailed introduction to this vast subject suitable for graduate students and others whose research has contact with crossed product $C*$-algebras. In addition to providing the basic definitions and results, the main focus of this book is the fine ideal structure of crossed products as revealed by the study of induced representations via the Green-Mackey-Rieffel machine. In particular, there is an in-depth analysis of the imprimitivity theorems on which Rieffel's theory of induced representations and Morita equivalence of $C*$-algebras are based. There is also a detailed treatment of the generalized Effros-Hahn conjecture and its proof due to Gootman, Rosenberg, and Sauvageot. This book is meant to be self-contained and accessible to any graduate student coming out of a first course on operator algebras. There are appendices that deal with ancillary subjects, which while not central to the subject, are nevertheless crucial for a complete understanding of the material. Some of the appendices will be of independent interest. To view another book by this author, please visit Morita Equivalence and Continuous-Trace $C*$-Algebras.
This book offers a comprehensive introduction by three of the leading experts in the field, collecting fundamental results and open problems in a single volume. Since Leavitt path algebras were first defined in 2005, interest in these algebras has grown substantially, with ring theorists as well as researchers working in graph C*-algebras, group theory and symbolic dynamics attracted to the topic. Providing a historical perspective on the subject, the authors review existing arguments, establish new results, and outline the major themes and ring-theoretic concepts, such as the ideal structure, Z-grading and the close link between Leavitt path algebras and graph C*-algebras. The book also presents key lines of current research, including the Algebraic Kirchberg Phillips Question, various additional classification questions, and connections to noncommutative algebraic geometry. Leavitt Path Algebras will appeal to graduate students and researchers working in the field and related areas, such as C*-algebras and symbolic dynamics. With its descriptive writing style, this book is highly accessible.
Mathematics for infinite dimensional objects is becoming more and more important today both in theory and application. Rings of operators, renamed von Neumann algebras by J. Dixmier, were first introduced by J. von Neumann fifty years ago, 1929, in [254] with his grand aim of giving a sound founda tion to mathematical sciences of infinite nature. J. von Neumann and his collaborator F. J. Murray laid down the foundation for this new field of mathematics, operator algebras, in a series of papers, [240], [241], [242], [257] and [259], during the period of the 1930s and early in the 1940s. In the introduction to this series of investigations, they stated Their solution 1 {to the problems of understanding rings of operators) seems to be essential for the further advance of abstract operator theory in Hilbert space under several aspects. First, the formal calculus with operator-rings leads to them. Second, our attempts to generalize the theory of unitary group-representations essentially beyond their classical frame have always been blocked by the unsolved questions connected with these problems. Third, various aspects of the quantum mechanical formalism suggest strongly the elucidation of this subject. Fourth, the knowledge obtained in these investigations gives an approach to a class of abstract algebras without a finite basis, which seems to differ essentially from all types hitherto investigated. Since then there has appeared a large volume of literature, and a great deal of progress has been achieved by many mathematicians.
This book is addressed to the researchers working in the theory of table algebras and association schemes. This area of algebraic combinatorics has been rapidly developed during the last decade. The volume contains further developments in the theory of table algebras. It collects several papers which deal with a classification problem for standard integral table algebras (SITA). More precisely, we consider SITA with a faithful non-real element of small degree. It turns out that such SITA with some extra conditions may be classified. This leads to new infinite series of SITA which has interesting properties. The last section of the book uses a part of obtained results in the classification of association schemes. This volume summarizes the research which was done at Bar-Ilan University in the academic year 1998/99.
This book provides, for the first time, a clear and unified exposition of the main techniques and results in operator algebras.
This book offers a comprehensive introduction to the general theory of C*-algebras and von Neumann algebras. Beginning with the basics, the theory is developed through such topics as tensor products, nuclearity and exactness, crossed products, K-theory, and quasidiagonality. The presentation carefully and precisely explains the main features of each part of the theory of operator algebras; most important arguments are at least outlined and many are presented in full detail.
Combining analysis, geometry, and topology, this volume provides an introduction to current ideas involving the application of

K-theory of operator algebras to index theory and geometry. In particular, the articles follow two main themes: the use of operator algebras to reflect properties of geometric objects and the application of index theory in settings where the relevant elliptic operators are invertible modulo a C^*-algebra other than that of the compact operators. The papers in this collection are the proceedings of the special sessions held at two AMS meetings: the Annual meeting in New Orleans in January 1986, and the Central Section meeting in April 1986. Jonathan Rosenberg's exposition supplies the best available introduction to Kasparov's KK-theory and its applications to representation theory and geometry. A striking application of these ideas is found in Thierry Fack's paper, which provides a complete and detailed proof of the Novikov Conjecture for fundamental groups of manifolds of non-positive curvature. Some of the papers involve Connes' foliation algebra and its K-theory, while others examine C^*-algebras associated to groups and group actions on spaces.

The main result of this original research monograph is the classification of C*-algebras of ordinary foliations of the plane in terms of a class of -trees. It reveals a close connection between some most recent developments in modern analysis and low-dimensional topology. It introduces noncommutative CW-complexes (as the global fibred products of C*-algebras), among other things, which adds a new aspect to the fast-growing field of noncommutative topology and geometry. The reader is only required to know basic functional analysis. However, some knowledge of topology and dynamical systems will be helpful. The book addresses graduate students and experts in the area of analysis, dynamical systems and topology.

This monograph deals with two aspects of the theory of elliptic genus: its topological aspect involving elliptic functions, and its representation theoretic aspect involving vertex operator super-algebras. For the second aspect, elliptic genera are shown to have the structure of modules over certain vertex operator super-algebras. The vertex operators corresponding to parallel tensor fields on closed Riemannian Spin Kähler manifolds such as Riemannian tensors and Kähler forms are shown to give rise to Virasoro algebras and affine Lie algebras. This monograph is chiefly intended for topologists and it includes accounts on topics outside of topology such as vertex operator algebras.

C^*-approximation theory has provided the foundation for many of the most important conceptual breakthroughs and applications of operator algebras. This book systematically studies (most of) the numerous types of approximation properties that have been important in recent years: nuclearity, exactness, quasidiagonality, local reflexivity, and others. Moreover, it contains user-friendly proofs, insofar as that is possible, of many fundamental results that were previously quite hard to extract from the literature. Indeed, perhaps the most important novelty of the first ten chapters is an earnest attempt to explain some fundamental, but difficult and technical, results as painlessly as possible. The latter half of the book presents related topics and applications--written with researchers and advanced, well-trained students in mind. The authors have tried to meet the needs both of students wishing to learn the basics of an important area of research as well as researchers who desire a fairly comprehensive reference for the theory and applications of C^*-approximation theory.