Logic And Set Theory With Applications 6th Edition

This is an introduction to logic and the axiomatization of set theory from a unique standpoint. Philosophical considerations, which are often ignored or treated casually, are here given careful consideration, and furthermore the author places the notion of inductively defined sets (recursive datatypes) at the center of his exposition resulting in a treatment of well established topics that is fresh and insightful. The presentation is engaging, but always great care is taken to illustrate difficult points. Understanding is also aided by the inclusion of many exercises. Little previous knowledge of logic is required of the reader, and only a background of standard undergraduate mathematics is assumed.

For a one-semester freshman or sophomore level course on the fundamentals of proof writing or transition to advanced mathematics course. Rather than teach mathematics and the structure of proofs simultaneously, this text first introduces logic as the foundation of proofs and then demonstrates how logic applies to mathematical topics. This method ensures that the students gain a firm understanding of how logic interacts with mathematics and empowers them to solve more complex problems in future math courses.

Rigorous coverage of logic and set theory for students of mathematics and philosophy.

This short textbook provides a succinct introduction to mathematical logic and set theory, which together form the foundations for the rigorous development of mathematics. It will be suitable for all mathematics undergraduates coming to the subject for the first time. The book is based on lectures given at the University of Cambridge and covers the basic concepts of logic: first order logic, consistency, and the completeness theorem, before introducing the reader to the fundamentals of axiomatic set theory. There are also chapters on recursive functions, the axiom of choice, ordinal and cardinal arithmetic and the incompleteness theorems. Dr Johnstone has included numerous exercises designed to illustrate the key elements of the theory and to provide applications of basic logical concepts to other areas of mathematics. Consequently the book, while making an attractive first textbook for those who plan to specialise in logic, will be particularly valuable for mathematics and computer scientists whose primary interests lie elsewhere.

Explores sets and relations, the natural number sequence and its generalization, extension of natural numbers to real numbers, logic, informal axiomatic mathematics, Boolean algebras, informal axiomatic set theory, several algebraic theories, and 1st-order theories.

While intuitionistic (or constructive) set theory IST has received a certain attention from mathematical logicians, so far as I am aware no book providing a systematic introduction to the subject has yet been published. This may be the case in part because, as a form of higher-order intuitionistic logic - the internal logic of a topos - IST has been chiefly developed in a tops-theoretic context. In particular, proofs of relative consistency with IST for mathematical assertions have been (implicitly) formulated in topos- or sheaf-theoretic terms, rather than in the framework of Heyting-algebra-valued models, the natural extension to IST of the well-known Boolean-valued models for classical set theory. In this book I offer a brief but systematic introduction to IST which develops the subject up to and including the use of Heyting-algebra-valued models in relative consistency proofs. I believe that IST, presented as it is in the familiar language of set theory, will appeal particularly to those logicians, mathematicians and philosophers who are unacquainted with the methods of topos theory.

This two-volume work bridges the gap between introductory expositions of logic (or set theory) and the research literature. It can be used as a text in an advanced undergraduate or beginning graduate course in mathematics, computer science, or philosophy. The volumes are written in a user-friendly lecture style that makes them equally effective for self-study or class use. Volume I includes formal proof techniques, applications of compactness (including nonstandard analysis), computability and its relation to the completeness phenonmenon, and the first presentation of a complete proof of Godel's 2nd incompleteness since Hilbert and Bernay's Grundlagen.

Problems in Set Theory, Mathematical Logic and the Theory of Algorithms by I. Lavrov & L. Maksimova is an English translation of the fourth edition of the most popular student problem book in mathematical logic in Russian. It covers major classical topics in proof theory and the semantics of propositional and predicate logic as well as set theory and computation theory. Each chapter begins with 1-2 pages of terminology and definitions that make the book self-contained. Solutions are provided. The book is likely to become an essential part of curricula in logic.

Michael Potter presents a comprehensive new philosophical introduction to set theory. Anyone wishing to work on the logical foundations of mathematics must understand set theory, which lies at its heart. Potter offers a thorough account of cardinal and ordinal arithmetic, and the various axiom candidates. He discusses in detail the project of set-theoretic reduction, which aims to interpret the rest of mathematics in terms of set theory. The key question here is how to deal with the paradoxes that bedevil set theory. Potter offers a strikingly simple version of the most widely accepted response to the paradoxes, which classifies sets by means of a hierarchy of levels. What makes the book unique is that it interweaves a careful presentation of the technical material with a penetrating philosophical critique. Potter does not merely expound the theory dogmatically but at every stage discusses in detail the reasons that can be offered for believing it to be true. Set Theory and its Philosophy is a key text for philosophy, mathematical logic, and computer science.

A mathematical introduction to the theory and applications of logic and set theory with an emphasis on writing proofs Highlighting the applications and notations of basic mathematical concepts within the framework of logic and set theory, A First Course in Mathematical Logic and Set Theory introduces how logic is used to prepare and structure proofs and solve more complex problems. The book begins with propositional logic, including two-column proofs and truth table applications, followed by first-order logic, which provides the structure for writing mathematical proofs. Set theory is then introduced and serves as the basis for defining relations, functions, numbers, mathematical induction, ordinals, and cardinals. The book concludes with a primer on basic model theory with applications to abstract algebra. A First Course in Mathematical Logic and Set Theory also includes: Section exercises designed to show the interactions between topics and reinforce the presented ideas and concepts Numerous examples that illustrate theorems and employ basic concepts such as Euclid’s lemma, the Fibonacci sequence, and unique factorization Coverage of important theorems including the well-ordering theorem, completeness theorem, compactness theorem, as well as the theorems of Löwenheim–Skolem, Burali-Forti, Hartogs, Cantor–Schröder–Bernstein, and König An excellent textbook for students studying the foundations of mathematics and mathematical proofs, A First Course in Mathematical Logic and Set Theory is also appropriate for readers preparing for careers in mathematics education or computer science. In addition, the book is ideal for introductory courses on mathematical logic and/or set theory and appropriate for upper-undergraduate transition courses with rigorous mathematical reasoning involving algebra, number theory, or analysis.

This book gives a rigorous yet 'physics-focused' introduction to mathematical logic that is geared towards natural science majors. We present the science major with a robust introduction to logic, focusing on the specific knowledge and skills that will unavoidably be needed in calculus topics and natural science topics in general (rather than taking a philosophical math fundamental oriented approach that is commonly found in mathematical logic textbooks).

The authors offer a clear, succinct and basic introduction to set theory and formal logic for linguists.

This book deals with two important branches of mathematics, namely, logic and set theory. Logic and set theory are closely related and play very crucial roles in the foundation of mathematics, and together produce several results in all of mathematics. The topics of logic and set theory are required in many areas of physical sciences, engineering, and technology. The book offers solved examples and exercises, and provides reasonable details to each topic discussed, for easy understanding. The book is designed for readers from various disciplines where mathematical logic and set theory play a crucial role. The book will be of interested to students and instructors in engineering, mathematics, computer science, and technology.

Descriptive set theory has been one of the main areas of research in set theory for almost a century. This text presents a largely balanced approach to the subject, which combines many elements of the different traditions. It includes a wide variety of examples, more than 400 exercises, and applications, in order to illustrate the general concepts and results of the theory.

This book explores and highlights the fertile interaction between logic and operator algebras, which in recent years has led to the resolution of several long-standing open problems on C*-algebras. The interplay between logic and operator algebras (C*-algebras, in particular) is relatively young and the author is at the forefront of this interaction. The deep level of scholarship contained in these pages is evident and opens doors to operator algebraists interested in learning about the set-theoretic methods relevant to their field, as well as to set-theorists interested in expanding their view to the non-commutative realm of operator algebras. Enough background is included from both subjects to make the book a convenient, self-contained source for students. A fair number of the exercises form an integral part of the text. They are chosen to widen and deepen the material from the corresponding chapters. Some other exercises serve as a warmup for the latter chapters.

Presents those methods of modern set theory most applicable to other areas of pure mathematics.

Set theory is the mathematics of infinity and part of the core curriculum for mathematics majors. This book blends theory and connections with other parts of mathematics so that readers can understand the place of set theory within the wider context. Beginning with the theoretical fundamentals, the author proceeds to illustrate applications to topology, analysis and combinatorics, as well as to pure set theory. Concepts such as Boolean algebras, trees, games, dense linear orderings, ideals, filters and club and stationary sets are also developed. Pitched specifically at undergraduate students, the approach is neither esoteric nor encyclopedic. The author, an experienced instructor, includes motivating examples and over 100 exercises designed for homework assignments, reviews and exams. It is appropriate for undergraduates as a course textbook or for self-study. Graduate students and researchers will also find it useful as a refresher or to solidify their understanding of basic set theory.

This must-read text presents the pioneering work of the late Professor Jacob (Jack) T. Schwartz on computational logic and set theory and its application to proof verification techniques, culminating in the ÆtnaNova system, a prototype computer program designed to verify the correctness of mathematical proofs presented in the language of set theory. Topics and features: describes in depth how a specific first-order theory can be exploited to model and carry out reasoning in branches of computer science and mathematics; presents an unique system for automated proof verification in large-scale software systems; integrates important proof-engineering issues, reflecting the goals of large-scale verifiers; includes an appendix showing formalized proofs of ordinals, of various properties of the transitive closure operation, of finite and transfinite induction principles, and of Zorn’s lemma.

This book is designed for readers who know elementary mathematical logic and axiomatic set theory, and who want to learn more about set theory. The primary focus of the book is on the independence proofs. Most famous among these is the independence of the Continuum Hypothesis (CH); that is, there are models of the axioms of set theory (ZFC) in which CH is true, and other models in which CH is false. More generally, cardinal exponentiation on the regular cardinals can consistently be anything not contradicting the classical theorems of Cantor and König. The basic methods for the independence proofs are the notion of constructibility, introduced by Gödel, and the method of forcing, introduced by Cohen. This book describes these methods in detail, verifi es the basic independence results for cardinal exponentiation, and also applies these methods to prove the independence of various mathematical questions in measure theory and general topology. Before the chapters on forcing, there is a fairly long chapter on "infi nitary combinatorics". This consists of just mathematical theorems (not independence results), but it stresses the areas of mathematics where set-theoretic topics (such as cardinal arithmetic) are relevant. There is, in fact, an interplay between infi nitary combinatorics and independence proofs. Infi nitary combinatorics suggests many set-theoretic questions that turn out to be independent of ZFC, but it also provides the basic tools used in forcing arguments. In particular, Martin's Axiom, which is one of the topics under infi nitary combinatorics, introduces many of the basic ingredients of forcing.

Set theory can be considered a unifying theory for mathematics. This book covers the fundamentals of the subject.

Studies in Logic and the Foundations of Mathematics, Volume 102: Set Theory: An Introduction to Independence Proofs offers an introduction to relative consistency proofs in axiomatic set theory, including combinatorics, sets, trees, and forcing. The book first tackles the foundations of set theory and infinitary combinatorics. Discussions focus on the Suslin problem, Martin's axiom, almost disjoint and quasi-disjoint sets, trees, extensionality and comprehension, relations, functions, and well-ordering, ordinals, cardinals, and real numbers. The manuscript then ponders on well-founded sets and easy consistency proofs, including relativization, absoluteness, reflection theorems, properties of well-founded sets, and induction and recursion on well-founded relations. The publication examines constructible sets, forcing, and iterated forcing. Topics include Easton forcing, general iterated forcing, Cohen model, forcing with partial functions of larger cardinality, forcing with finite partial functions, and general extensions. The manuscript is a dependable source of information for mathematicians and researchers interested in set theory.

This is an extensively revised edition of Mr. Quine's introduction to abstract set theory and to various axiomatic systematizations of the subject. The treatment of ordinal numbers has been strengthened and much simplified, especially in the theory of transfinite recursions, by adding an axiom and reworking the proofs. Infinite cardinals are treated anew in clearer and fuller terms than before. Improvements have been made all through the book; in various instances a proof has been shortened, a theorem strengthened, a space-saving lemma inserted, an obscurity clarified, an error corrected, a historical omission supplied, or a new event noted.

Provability, Computability and Reflection

"There are many textbooks available for a so-called transition course from calculus to abstract mathematics. I have taught this course several times and always find it problematic. The Foundations of Mathematics (Stewart and Tall) is a horse of a different color. The writing is excellent and there is actually some useful mathematics. I definitely like this book."--The Bulletin of Mathematics Books

This book is especially useful for students who are about to finish high school and want to properly prepare in mathematics to start college or university. The book is also suitable for those students who have already started higher education. Many of these students still face barriers in learning more complex mathematical concepts and their adequate application. A flaw in understanding basic concepts is what makes them reluctant to learn more.Since the objective is to help students achieve an adequate understanding of basic concepts, this book was written in plain language to make the process of acquiring mathematical knowledge a friendly, enjoyable, and accessible one even for those students who dislike mathematics. The simplicity of language does not sacrifice the rigor or depth of the study offered in the next pages, which guarantees adequate management of concepts. Students are not required to have math skills; the only requirement is to be interested in learning. In the three chapters that make up this text, we address the building blocks of Calculus or Mathematical Analysis. We start with Propositional Logic, which provides the language, logical reasoning, and training on how to properly address mathematical proofs. This chapter is followed by an introduction to Set Theory where we develop a body of concepts that are used in the definition of many fundamental concepts of Calculus such as Function. This book ends with the study of the real number system which is addressed from an axiomatic approach. Throughout the book, we offer examples to illustrate all the concepts that we discuss. Additionally, a set of 484 exercises are proposed to be solved by students. The answers to those exercises are offered at the end of each chapter. Thus, students can check their progress in learning the concepts discussed. The level of difficulty of such exercises varies from the most elementary level to a moderate level since the main objective of this book is to help students to properly learn these basic concepts and not to test their mathematical skills.

This is an introductory undergraduate textbook in set theory. In mathematics these days, essentially everything is a set. Some knowledge of set theory is necessary part of the background everyone needs for further study of mathematics. It is also possible to study set theory for its own interest--it is a subject with intruiging results anout simple objects. This book starts with material that nobody can do without. There is no end to what can be learned of set theory, but here is a beginning.

Set theory, logic and category theory lie at the foundations of mathematics, and have a dramatic effect on the mathematics that we do, through the Axiom of Choice, Gödel's Theorem, and the Skolem Paradox. But they are also rich mathematical theories in their own right, contributing techniques and results to working mathematicians such as the Compactness Theorem and module categories. The book is aimed at those who know some mathematics and want to know more about its building blocks. Set theory is first treated naively an axiomatic treatment is given after the basics of first-order logic have been introduced. The discussion is su pported by a wide range of exercises. The final chapter touches on philosophical issues. The book is supported by a World Wibe Web site containing a variety of supplementary material.

Mathematical Reasoning: Writing and Proof is a text for the ?rst college mathematics course that introduces students to the processes of constructing and writing proofs and focuses on the formal development of mathematics. The primary goals of the text are to help students: Develop logical thinking skills and to develop the ability to think more abstractly in a proof oriented setting; develop the ability to construct and write mathematical proofs using standard methods of mathematical proof including direct proofs, proof by contradiction, mathematical induction, case analysis, and counterexamples; develop the ability to read and understand written mathematical proofs; develop talents for creative thinking and problem solving; improve their quality of communication in mathematics. This includes improving writing techniques, reading comprehension, and oral communication in mathematics; better understand the nature of mathematics and its language. Another important goal of this text is to provide students with material that will be needed for their further study of mathematics. Important features of the book include: Emphasis on writing in mathematics; instruction in the process of constructing proofs; emphasis on active learning.There are no changes in content between Version 2.0 and previous versions of the book. The only change is that the appendix with answers and hints for selected exercises now contains solutions and hints for more exercises.

This lively introductory text exposes the student in the humanities to the world of discrete mathematics. A problem-solving based approach grounded in the ideas of George Pólya are at the heart of this book. Students learn to handle and solve new problems on their own. A straightforward, clear writing style and well-crafted examples with diagrams invite the students to develop into precise and critical thinkers. Particular attention has been given to the material that some students find challenging, such as proofs. This book illustrates how to spot invalid arguments, to enumerate possibilities, and to construct probabilities. It also presents case studies to students about the possible detrimental effects of ignoring these basic principles. The book is invaluable for a discrete and finite mathematics course at the freshman undergraduate level or for self-study since there are full solutions to the exercises in an appendix. "Written with clarity, humor and relevant real-world examples, Basic Discrete Mathematics is a wonderful introduction to discrete mathematical reasoning."- Arthur Benjamin, Professor of Mathematics at Harvey Mudd College, and author of The Magic of Math

Keith Devlin. You know him. You've read his columns in MAA Online, you've heard him on the radio, and you've seen his popular mathematics books. In between all those activities and his own research, he's been hard at work revising Sets, Functions and Logic, his standard-setting text that has smoothed the road to pure mathematics for legions of undergraduate students. Now in its third edition, Devlin has fully reworked the book to reflect a new generation. The narrative is more lively and less textbook-like. Remarks and asides link the topics presented to the real world of students' experience. The chapter on complex numbers and the discussion of formal symbolic logic are gone in favor of more exercises, and a new introductory chapter on the nature of mathematics--one that motivates readers and sets the stage for the challenges that lie ahead. Students crossing the bridge from calculus to higher mathematics need and deserve all the help they can get. Sets, Functions, and Logic, Third Edition is an affordable little book that all of your transition-course students not only can afford, but will actually read...and enjoy...and learn from. About the Author Dr. Keith Devlin is Executive Director of Stanford University's Center for the Study of Language and Information and a Consulting Professor of Mathematics at Stanford. He has written 23 books, one interactive book on CD-ROM, and over 70 published research articles. He is a Fellow of the American Association for the Advancement of Science, a World Economic Forum Fellow, and a former member of the Mathematical Sciences Education Board of the National Academy of Sciences,. Dr. Devlin is also one of the world's leading popularizers of mathematics. Known as "The Math Guy" on NPR's Weekend Edition, he is a frequent contributor to other local and national radio and TV shows in the US and Britain, writes a monthly

column for the Web journal MAA Online, and regularly writes on mathematics and computers for the British newspaper The Guardian.
What is a number? What is infinity? What is continuity? What is order? Answers to these fundamental questions obtained by late nineteenth-century mathematicians such as Dedekind and Cantor gave birth to set theory. This textbook presents classical set theory in an intuitive but concrete manner. To allow flexibility of topic selection in courses, the book is organized into four relatively independent parts with distinct mathematical flavors. Part I begins with the Dedekind–Peano axioms and ends with the construction of the real numbers. The core Cantor–Dedekind theory of cardinals, orders, and ordinals appears in Part II. Part III focuses on the real continuum. Finally, foundational issues and formal axioms are introduced in Part IV. Each part ends with a postscript chapter discussing topics beyond the scope of the main text, ranging from philosophical remarks to glimpses into landmark results of modern set theory such as the resolution of Lusin's problems on projective sets using determinacy of infinite games and large cardinals. Separating the metamathematical issues into an optional fourth part at the end makes this textbook suitable for students interested in any field of mathematics, not just for those planning to specialize in logic or foundations. There is enough material in the text for a year-long course at the upper-undergraduate level. For shorter one-semester or one-quarter courses, a variety of arrangements of topics are possible. The book will be a useful resource for both experts working in a relevant or adjacent area and beginners wanting to learn set theory via self-study.
This book is intended as an undergraduate senior level or beginning graduate level text for mathematical logic. There are virtually no prere quisites, although a familiarity with notions encountered in a beginning course in abstract algebra such as groups, rings, and fields will be useful in providing some motivation for the topics in Part III. An attempt has been made to develop the beginning of each part slowly and then to gradually quicken the pace and the complexity of the material. Each part ends with a brief introduction to selected topics of current interest. The text is divided into three parts: one dealing with set theory, another with computable function theory, and the last with model theory. Part III relies heavily on the notation, concepts and results discussed in Part I and to some extent on Part II. Parts I and II are independent of each other, and each provides enough material for a one semester course. The exercises cover a wide range of difficulty with an emphasis on more routine problems in the earlier sections of each part in order to familiarize the reader with the new notions and methods. The more difficult exercises are accompanied by hints. In some cases significant theorems are devel oped step by step with hints in the problems. Such theorems are not used later in the sequence.